JENN

Training and Consultancy

The path to enlightened education

SUBJECT: MATHEMATICS

GRADE 12

2026 LAST PSUH

QUESTION PAPER BANK

PAPER 1

&

PAPER 2

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INFORMATION SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1+ni) \qquad A = P(1-ni) \qquad A = P(1-i)^n \qquad A = P(1+i)^n$$

$$T_n = a + (n-1)d \qquad S_n = \frac{n}{2}[2a + (n-1)d]$$

$$T_n = ar^{n-1} \qquad S_n = \frac{a(r^n - 1)}{r-1} \; ; \quad r \neq 1 \qquad S_\infty = \frac{a}{1-r} \; ; \; -1 < r < 1$$

$$F = \frac{x[(1+i)^n - 1]}{i} \qquad P = \frac{[1-(1+i)^{-n}]}{i}$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad \mathrm{M}\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c \qquad y - y_1 = m(x - x_1) \qquad m = \frac{y_2 - y_1}{x_2 - x_1} \qquad m = \tan\theta$$

$$(x-a)^2 + (y-b)^2 = r^2$$

$$In\ \Delta ABC: \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \qquad a^2 = b^2 + c^2 - 2bc.\cos A \qquad area\ \Delta ABC = \frac{1}{2}ab.\sin C$$

$$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta \qquad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$$

$$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta \qquad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases} \qquad \sin 2\alpha = 2\sin\alpha.\cos\alpha$$

$$\bar{x} = \frac{\sum fx}{n} \qquad \sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)} \qquad P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$\hat{y} = a + bx \qquad b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$$

SECTION A: PAPER 1

ALGEBRA, EQUATIONS AND INEQUALITIES

Paper A

QUESTION 1

1.1 Solve for x:

1.1.1 $3x^2 + 5x = 0$ (2)

1.1.2 $4x^2 + 3x - 5 = 0$ (answers correct to TWO decimal places) (3)

1.1.3 $(x-1)^2 - 9 \geq 0$ (4)

1.1.4 $5^{2x} - 5^x = 0$ (4)

1.1.5 $\dfrac{x}{\sqrt{20-x}} = 1$ (5)

1.2 Solve for x and y simultaneously:

$x + y = 9$ and $2x^2 - y^2 = 7$ (5)

1.3 Given: $P = (1-a)$ and $T = (1+a)(1+a^2)(1+a^4)..(1+a^{512})$

Determine the value of $P \times T$ in terms of a. (3)

[26]

Paper B

QUESTION 1

1.1 Solve for x:

1.1.1 $x^2 + x - 12 = 0$ (3)

1.1.2 $3x^2 - 2x = 6$ (answers correct to TWO decimal places) (4)

1.1.3 $\sqrt{2x+1} = x - 1$ (4)

1.1.4 $x^2 - 3 > 2x$ (4)

1.2 Solve for x and y simultaneously:

$x + 2 = 2y$ and $\dfrac{1}{x} + \dfrac{1}{y} = 1$ (5)

1.3 Given: $2^{m+1} + 2^m = 3^{n+2} - 3^n$ where m and n are integers.

Determine the value of $m + n$. (4)

[24]

Paper C

QUESTION 1

1.1 Solve for x:

1.1.1 $(3x-6)(x+2)=0$ (2)

1.1.2 $2x^2-6x+1=0$ (correct to TWO decimal places) (3)

1.1.3 $x^2-90>x$ (4)

1.1.4 $x-7\sqrt{x}=-12$ (4)

1.2 Solve for x and y simultaneously:

$$2x-y=2$$
$$xy=4$$ (5)

1.3 Show that $2.5^n-5^{n+1}+5^{n+2}$ is even for all positive integer values of n. (3)

1.4 Determine the values of x and y if: $\frac{3^{y+1}}{32}=\sqrt{96^x}$ (4)

[25]

Paper D

QUESTION 1

1.1 Solve for x:

1.1.1 $3x(2x+1)=0$ (2)

1.1.2 $5x^2+3x-1=0$ (correct to TWO decimal places) (3)

1.1.3 $2x^2+6x-8\geq 0$ (3)

1.1.4 $\sqrt{3x+13}=x+1$ (4)

1.1.5 $3^{2x+2}+8.3^x-1=0$ (4)

1.2 Solve for x and y simultaneously:

$x-2y-3=0$ and $x^2-3xy+y^2=11$ (6)

1.3 Given: $f(x)=x^2-5x+c$

Determine the value of c if the solution for $f(x)=0$ is $\frac{(5\pm\sqrt{41})}{2}$. (3)

[25]

Paper E

QUESTION 1

1.1 Solve for x:

1.1.1 $x^2 - 121 = 0$ (3)

1.1.2 $x^2 + 3 = 5x$ (Correct to **TWO** decimal places) (4)

1.1.3 $(x-1)(x-5) \leq 12$ (4)

1.1.4 $2^x + 2^{x+2} = 40$ (4)

1.1.5 $\sqrt{10-x^2} - x + 2 = 0$ (5)

1.2 Solve simultaneously for x and y:

$y - x = 2$

$x^2 + 2xy - 4 = 0$ (5)

1.3 Given: $f(x) = x(x+a) - b$ and $g(x) = 2x^2 + b - dx$, where $a \neq 0$ and $b \neq 0$.

If $x - a$ is a factor of $f(x)$ and if $x - b$ is a factor of $g(x)$,

prove that: $d - 1 = 4a^2$ (5)

[30]

Paper F

QUESTION 1

1.1 Solve for x:

1.1.1 $x(x+6) = 0$ (2)

1.1.2 $3x^2 + 8x = -2$ (correct tot TWO decimal places) (4)

1.1.3 $x^2 - 64 \leq 0$ (3)

1.1.4 $\sqrt{x+5} + 1 = x$ (5)

1.2 Solve simultaneously for x and y in the following equations:

$6x + 5xy - 5y = 8$ (6)

$x + y = 2$

[20]

Paper G

QUESTION 1

1.1 Solve for x:

1.1.1 $(x+3)(2-5x)=0$ (2)

1.1.2 $7x^2+5x-8=0$ (correct to TWO decimal places) (3)

1.1.3 $\frac{7}{x^2-2x-8}>0$ (4)

1.1.4 $3^{x+2}=42-5.3^x$ (3)

1.1.5 $x-\sqrt{5x-1}=5$ (5)

1.2 Solve for x and y simultaneously:

$x-y=3$
$x^2-xy=2y^2+7$ (6)

1.3 Prove that the roots of $2x^2+px-p^2=0$ are rational for all rational values of p. (3)

[26]

Paper H

QUESTION 1

1.1 Solve for x:

1.1.1 $x^2+2x+1=0$ (2)

1.1.2 $x(5x-3)=1$ (correct to TWO decimal places) (4)

1.1.3 $2x+3>x^2$ (4)

1.1.4 $\sqrt{7x-12}-x=0$ (4)

1.1.5 $\left(\frac{1}{6}\right)^{3x+2}.216^{3x}=\frac{1}{216}$ (4)

1.2 Solve simultaneously for x and y:

$x+y=3$
$x^2+2y^2=18$ (5)

1.3 Given: $\left(\sqrt{5}\right)^x-\left(\sqrt[3]{2}\right)^y=17$; where x and $y\in\mathbb{N}$, determine a value for $P=(x\times y)$. (4)

[27]

Paper I

QUESTION 1

1.1 Solve for x:

1.1.1 $(3x-6)(x+5)=0$ (2)

1.1.2 $7x^2+4x-9=0$ (correct to TWO decimal places) (3)

1.1.3 $(3-x)(x+7)<0$ (3)

1.1.4 $3\sqrt{x+3}-5=x$ (5)

1.2 Solve for x and y:

$y=3x+2$

$\frac{1}{y}+\frac{3}{x}=-4$ (5)

1.3 If, $\frac{2^x}{16}=\frac{3^y}{3}$ determine the value of $x+y$ (4)

[22]

Paper J

QUESTION 1

1.1 Solve for x:

1.1.1 $(x+2)(x-2)=5$ (2)

1.1.2 $\frac{1}{2}x^2-2x-1=0$ (correct to TWO decimal places) (3)

1.1.3 $5<x(7-2x)$ and $x>2$ (5)

1.1.4 $3^x-1=\sqrt{2.3^{x+1}+10}$ (4)

1.2 The roots of a quadratic equation $x^2+bx+c=0$ are $x=\pm\sqrt{p^2-2p+1}$

1.2.1 For which value(s) of p will the roots be real and equal? (2)

1.2.2 Hence, determine the value(s) of b and c for the value of p in QUESTION 1.2.1 (3)

1.3 Solve for x and y in the following simultaneous equations:

$\frac{4^y}{2^x}=0,5$ and $x^2+y^2=29$ (7)

[26]

QUESTION 2

2.1 Consider the geometric series: $4+2+1+\frac{1}{2}+...$

2.1.1 Does this series converge? Justify your answer. (2)

2.1.2 Calculate S_∞. (2)

2.2 Given: $\sum_{p=k}^{10} 3^{p-1} = 29\,520$. Calculate the value of k. (5)
[9]

QUESTION 3

3.1 Consider the quadratic number pattern: 3 ; 7 ; 12 ; …

3.1.1 Show that the general term of this number pattern is given by $T_n = \frac{1}{2}n^2 + \frac{5}{2}n$. (3)

3.1.2 What number must be added to T_{n-1} so that $T_n = 13\,527$? (4)

3.2 Given an arithmetic sequence with $T_1 = 8$ and $T_2 = 11$.

3.2.1 Calculate the value of n if $T_n = 41$. (3)

3.2.2 A new arithmetic sequence P is formed using the term position and the term value of the given arithmetic sequence.
For the new sequence, $P_8 = 1$, $P_{11} = 2$ and so forth.

(a) Write down the value of P_{41}. (1)

(b) Calculate the value of the first term of the new arithmetic sequence. (4)
[15]

Paper B

QUESTION 2

2.1 Given the infinite geometric series: $(t+10)+(t-2)+(t+4)+\ldots$

2.1.1 Show that $t=-2$ (3)

2.1.2 Calculate the value of T_{25}. Write your answer in the form $T_n = b^x$ (3)

2.1.3 Calculate the sum of the infinite series. (2)

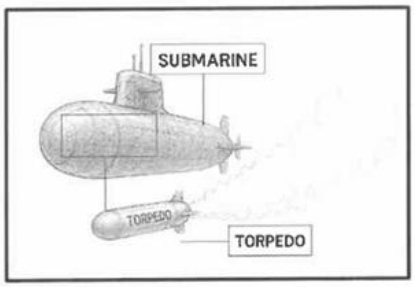
2.2 Given $\sum_{p=k}^{117}(4p-1)=26\,675$

2.2.1 Write down the difference between T_6 and T_{14} (2)

2.2.2 Calculate the value of k. (5)
[15]

QUESTION 3

The depth of a torpedo below sea level forms a quadratic pattern, where 0 metres is at sea level. A submarine tracked a torpedo in one-second intervals.



The depth (in metres) that the torpedo reached is given in the table below.

	Depth (in metres)
At the end of the first second	36
At the end of the first 2 seconds	71
At the end of the first 3 seconds	104

3.1 Calculate the depth of the torpedo at the end of the first 5 seconds. (2)

3.2 Show that the depth of the torpedo at the end of n seconds was $T_n = -n^2 + 38n - 1$ (3)

3.3 Calculate the maximum depth that the torpedo reached. (3)

3.4 After how many seconds was the torpedo at 104 m below sea level for the second time? (2)
[10]

Paper C

QUESTION 2

2.1 The first term of a geometric series is 14 and the 6^{th} term is 448.

2.1.1 Calculate the value of the constant ratio, r. (2)

2.1.2 Determine the number of consecutive terms that must be added to the first 6 terms of the series in order to obtain a sum of 114 674. (4)

2.1.3 If the first term of another series is 448 and the 6^{th} term is 14, calculate the sum to infinity of the new series. (3)

2.2 If $\sum_{p=0}^{k}\left(\frac{1}{3}p+\frac{1}{6}\right)=20\frac{1}{6}$, determine the value of k. (5)

[14]

QUESTION 3

It is given that the general term of a quadratic number pattern is $T_n = n^2 + bn + 9$ and the first term of the first differences is 7.

3.1 Show that $b = 4$. (2)

3.2 Determine the value of the 60^{th} term of this number pattern. (2)

3.3 Determine the general term for the sequence of first differences of the quadratic number pattern. Write your answer in the form $T_p = mp + q$. (3)

3.4 Which TWO consecutive terms in the quadratic number pattern have a first difference of 157? (3)

[10]

Paper D

QUESTION 2

2.1 Given the arithmetic series: $5 + 7 + 9 + \ldots + 93$

2.1.1 Determine the general term of the series, T_n, in the form $T_n = pn + q$. (2)

2.1.2 The given series represents the number of kilometres that an athlete ran each week in preparation for an ultramarathon. The athlete ran 93 km in the last week of the training programme. How long, in weeks, was the training programme? (2)

2.1.3 The athlete used this opportunity to raise funds for her high school. The community sponsored her R10 for each kilometre run during the training programme. Calculate the total amount that the athlete raised for her school. (3)

2.2 The general term of a geometric sequence is $T_n = 2^{n+2}$

2.2.1 Write down:

(a) The first term (1)

(b) The common ratio (1)

2.2.2 Calculate T_{20} (Write your answer as a power of 4.) (2)

2.2.3 Calculate $\sum_{n=1}^{\infty} \frac{1}{T_n}$ (3)

2.2.4 Consider the first 21 terms of the sequence $T_n = 2^{n+2}$. Calculate the sum of the terms in this sequence that are not powers of 4. (4)

[18]

QUESTION 3

Given the quadratic sequence: 14 ; 9 ; 6 ; 5 ; ...

3.1 Show that the general term of this sequence is $T_n = n^2 - 8n + 21$. (3)

3.2 Two consecutive terms of the quadratic sequence have a difference of 33. Calculate the value of the larger term. (3)

3.3 The value of m is added to each term in the quadratic sequence. Determine the values of m for which only the terms between T_1 and T_7 of the quadratic sequence will have negative values. (3)

[9]

Paper E

QUESTION 2

2.1 Given the quadratic sequence: 321 ; 290 ; 261 ; 234 ; ....

2.1.1 Write down the values of the next TWO terms of the sequence. (2)

2.1.2 Determine the general term of the sequence in the form $T_n = an^2 + bn + c$. (4)

2.1.3 Which term(s) of the sequence will have a value of 74? (4)

2.1.4 Which term in the sequence has the least value? (2)

2.2 Given the geometric series: $\frac{5}{8} + \frac{5}{16} + \frac{5}{32} + ... = K$

2.2.1 Determine the value of K if the series has 21 terms. (3)

2.2.2 Determine the largest value of n for which $T_n > \frac{5}{8192}$ (4)

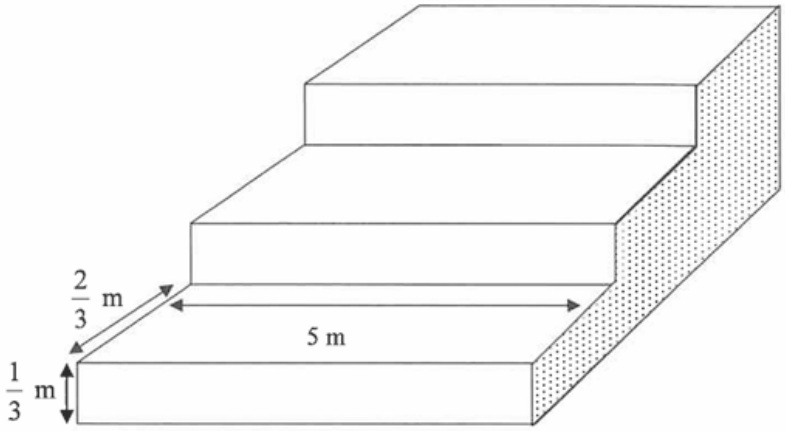
[19]

QUESTION 3

3.1 Without using a calculator, determine the value of: $\sum_{y=3}^{10} \frac{1}{y-2} - \sum_{y=3}^{10} \frac{1}{y-1}$ (3)

3.2 A steel pavilion at a sports ground comprises of a series of 12 steps, of which the first 3 are shown in the diagram below.

Each step is 5 m wide. Each step has a rise of $\frac{1}{3}$ m and has a tread of $\frac{2}{3}$ m, as shown in the diagram below.



The open side (shaded on sketch) on each side of the pavilion must be covered with metal sheeting. Calculate the area (in m^2) of metal sheeting needed to cover both open sides. (6)

[9]

Paper F

QUESTION 2

2.1 Consider the following quadratic pattern: x ; 17 ; y ; 57 ; 86.

Determine the values of x and y. (5)

2.2 An athlete trains by running 600 m on the first day. Thereafter, she increases the distance by 300 m every day. (This is a theoretic scenario.)

2.2.1 Calculate the distance she runs on the 15^{th} day. (3)

2.2.2 What is the total distance, in km, that she has run at the end of the 15^{th} day? (3)

2.2.3 In order to participate in the Comrades Marathon in 6 months (180 days), she must complete a qualifying race of 42 km. If she continues her pattern of training, will she have sufficient time to train for the 42 km qualifier and thus be able to participate in the Comrades? Show ALL calculations. (3)

2.3 Use an appropriate formula to calculate:

$$\sum_{n=1}^{21} (101-5n) \quad (3)$$

2.4 Given: A convergent geometric series with first term $T_1 = a$ and $S_\infty = p$ where $p > 0$.

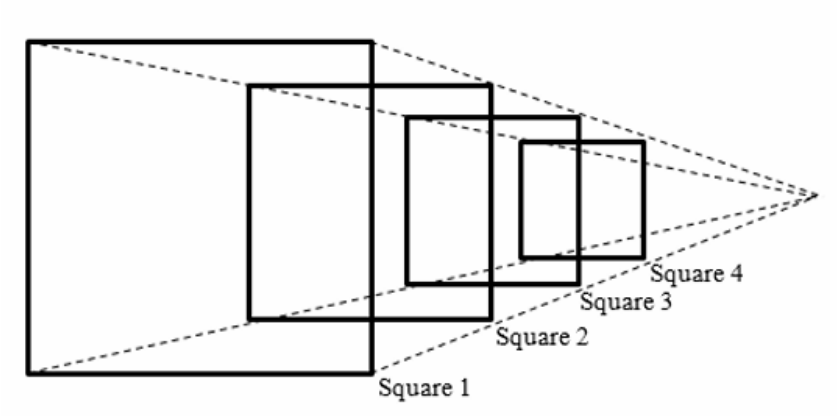
2.4.1 Show that $a \in (0\ ;\ 2p)$. (5)

2.4.2 Determine the value of the constant ratio if $a = \frac{p}{4}$. (3)
[25]

Paper G

QUESTION 2

2.1 Given the following geometric series: $36 + 18 + 9 + \frac{9}{2} + \ldots$ representing the areas of the squares in cm^2, as drawn below.



2.1.1 Does the series converge? Justify your answer. (2)

2.1.2 Calculate S_∞. (2)

2.1.3 Which square will have a side length of $\frac{3}{8}$ cm? (3)

2.1.4 Calculate the sum of the diagonals of the first ten squares. (4)

2.2 The seventh term of an arithmetic sequence is 4 and the twelfth term is 14.

2.2.1 Determine the common difference and the first term of the sequence. (3)

2.2.2 Which term of the sequence will be the additive inverse of the first term? (2)

2.3 Given: $\sum_{p=k}^{100}(4p-1) = 19995$, determine the value of k. (5)

[21]

QUESTION 3

3.1 Consider the quadratic number pattern: 3 ; 12 ; 33 ; 66 ; …

3.1.1 Write down the value of the next term of the quadratic number pattern. (2)

3.1.2 If the general term of the quadratic pattern is $T_n = 6n^2 - 9n + 6$, determine the value of $T_9 - T_5$. (2)

3.2 The following information of a quadratic pattern is given:

- $T_n = an^2 + bn + 1$
- $T_4 = 27$
- $T_3 - T_2 = 8$

Determine the values of a and b. (5)
[9]

Paper H

QUESTION 2

2.1 Given the arithmetic series: $6 + 1 - 4 - 9 \ldots\ldots$

2.1.1 Write down the value of the next term of the arithmetic series. (1)

2.1.2 Calculate: $6 + 1 - 4 - 9 \ldots\ldots\ldots - 239$ (5)

2.2 Consider a quadratic pattern: $-9 ; -5 ; x ; 15 ; \ldots$

2.2.1 Calculate the value of x. (3)

2.2.2 If the value of $x = 3$, determine the n^{th} term of the number pattern. (4)

2.2.3 Explain why all the terms of this quadratic pattern are odd numbers. (2)
[15]

QUESTION 3

3.1 Given: $\sum_{k=1}^{\infty} 4.p^{1-k} = 6$

3.1.1 Calculate the value of p. (4)

3.1.2 Hence, write down the first three terms of the series. (1)

3.2 On a particular day, a grade 12 learner from Dinaledi High School watched a video about number patterns on YouTube.

- At 1 p.m. he shared the link for the video with 5 of his friends.
- At 2 p.m. each of these 5 friends shared the link with 5 other friends.
- Then at 3 p.m., each of those 5 friends shared it again with 5 different people.

If this pattern continues in the same way:

3.2.1 Determine how many people will receive the link at exactly 4 p.m. (2)

3.2.2 Determine the total number of people who would have received the link by 11 p.m. (3)

[10]

Paper I

QUESTION 2

2.1 Consider the quadratic number pattern: $-20\ ;\ -9\ ;\ 0\ ;\ 7\ ;...$

2.1.1 Determine the n^{th} term . (4)

2.1.2 Determine the position and the value of the term with the highest value. (3)

[7]

QUESTION 3

3.1 Given the following arithmetic sequence: $13\ ;\ 8\ ;\ 3\ ;\ ..$

3.1.1 Determine the value of the 50^{th} term. (3)

3.1.2 Calculate the sum of the first fifty terms. (2)

3.2 Prove that: $a+a+d+a+2d+...\text{ (to } n\text{ terms)} = \frac{n}{2}\left[2a+(n-1)d\right]$ (4)

3.3 Consider the geometric series: $3+m+\frac{m^2}{3}+\frac{m^3}{9}+...$

3.3.1 For which value(s) of m will the series converge? (3)

3.3.2 It is given that: $3+m+\frac{m^2}{3}+\frac{m^3}{9}+...=\frac{27}{7}$

Calculate the value of m (3)

3.4 Determine the value of n if:

$$\sum_{r=1}^{n} 5.2^{1-r} = \frac{630}{64}$$ (6)

[21]

Paper J

QUESTION 2

2.1 The first term of an arithmetic series is 7. The common difference of this series is 5 and the series contains 20 terms.

2.1.1 Calculate the sum of this series. (2)

2.1.2 The original arithmetic series is extended to 75 terms. The sum of these 75 terms is 14 400. Using sigma notation, write down an equation for the sum of the terms added to the original series. (4)

2.2 The sequence of the first differences of a quadratic pattern is: 1 ; 3 ; 5 ; …

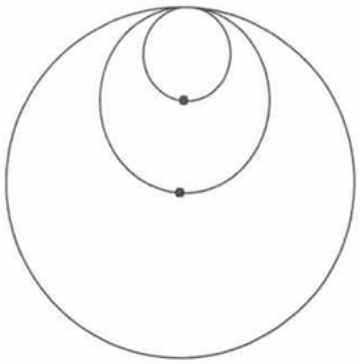
2.2.1 If T_{99} of this quadratic pattern is 9 632, calculate the value of T_{98}. (3)

2.2.2 If it is further given that the third term of the quadratic pattern is 32, determine the general term, T_n, of the quadratic pattern. (5)

[14]

QUESTION 3

A circle with radius 6 cm is drawn.
A second, smaller circle is drawn through the centre of the first circle and also touches the first circle internally, as shown in the diagram.
A third, smaller circle is drawn through the centre of the second circle and touches the second circle internally. The process of drawing circles continues and forms a geometric pattern.



3.1 Write down the radius of the 3rd circle. (2)

3.2 Calculate the sum of the areas of the first 10 circles. (4)

3.3 Which circle has a diameter of $\frac{3}{128}$ cm? (4)

[10]

FUNCTIONS AND GRAPHS

Paper A

QUESTION 4

Given: $g(x) = \frac{1}{x-1} + 2$

4.1 Write down the equations of the asymptotes of g. (2)

4.2 Draw a graph of g, indicating any intercepts with the axes and asymptotes. (4)

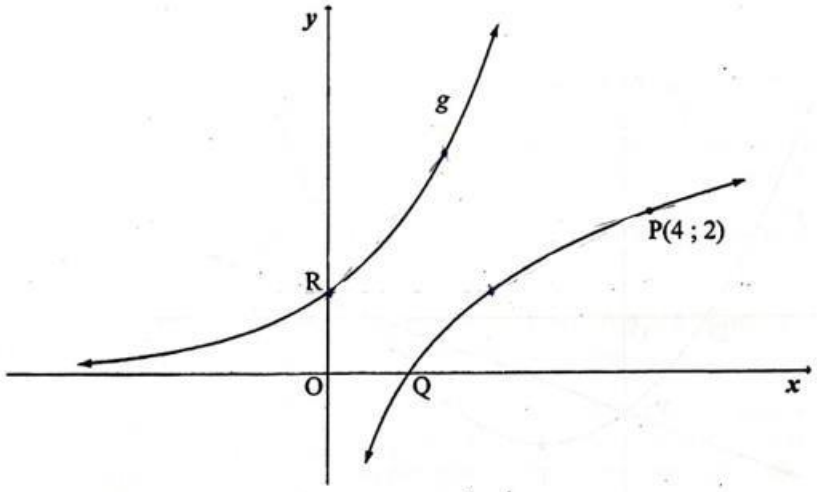
4.3 Determine the values of x where $g(x) > 0$. (2)

4.4 Determine the equation of the axis of symmetry of g which has a negative gradient. (2)

[10]

QUESTION 5

In the diagram, the graphs of $f(x) = \log_a x$ and g are drawn. Graph g is the reflection of f in the line $y = x$. Graph f passes through the point P(4 ; 2). Q is the x-intercept of f and R is the y-intercept of g.



5.1 Write down the coordinates of P', the image of P on g. (2)

5.2 Show that $a = 2$. (2)

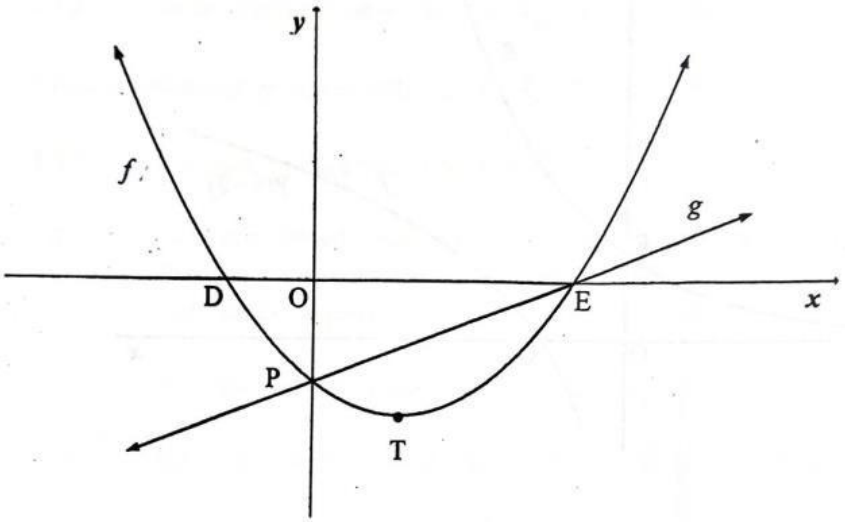
5.3 Write down the equation of g in the form $y = \ldots$ (1)

5.4 T is a point on f in the first quadrant where TR is parallel to the x-axis. Calculate the area of $\Delta RTP'$. (4)

[9]

QUESTION 6

The graphs of $f(x) = x^2 - 2x - 3$ and $g(x) = mx + c$ are drawn below. D and E are the x-intercepts and P is the y-intercept of f. The turning point of f is T(1 ; – 4). The graphs of f and g intersect at P and E.



6.1 Write down the range of f. (1)

6.2 Calculate the coordinates of D and E. (3)

6.3 Determine the equation of g. (2)

6.4 Write down the values of x for which $f(x) - g(x) > 0$. (2)

6.5 Determine the maximum vertical distance between h and g if $h(x) = -f(x)$ for $x \in [-2 ; 3]$. (5)

6.6 Given: $k(x) = g(x) - n$.

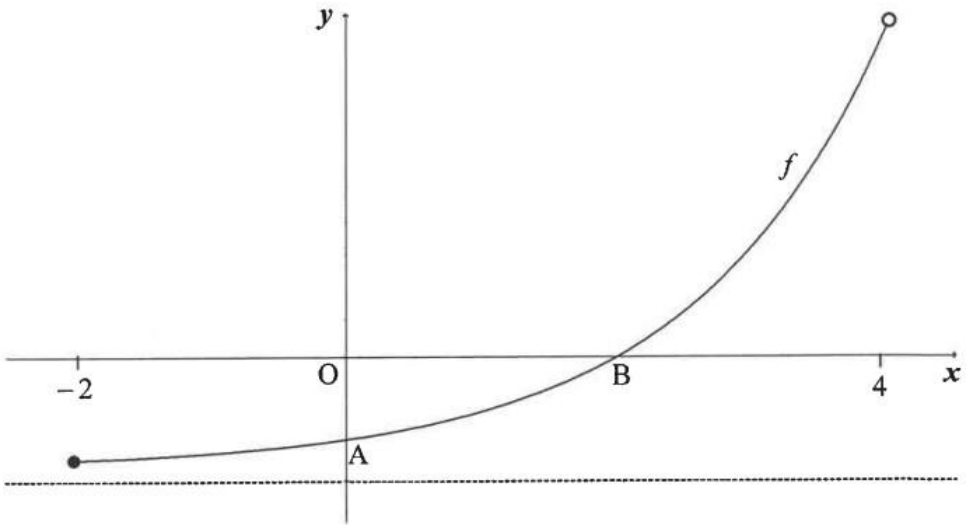
Determine n if k is a tangent to f. (5)

[18]

Paper B

QUESTION 4

Sketched below is the graph of $f(x) = 2^x - 4$ for $x \in [-2 ; 4)$.
A and B are respectively the y- and x-intercepts of f.



4.1 Write down the equation of the asymptote of f. (1)

4.2 Determine the coordinates of B. (2)

4.3 Determine the equation of k, a straight line passing through A and B in the form $k(x) = \ldots$ (3)

4.4 Calculate the vertical distance between k and f at $x = 1$ (3)

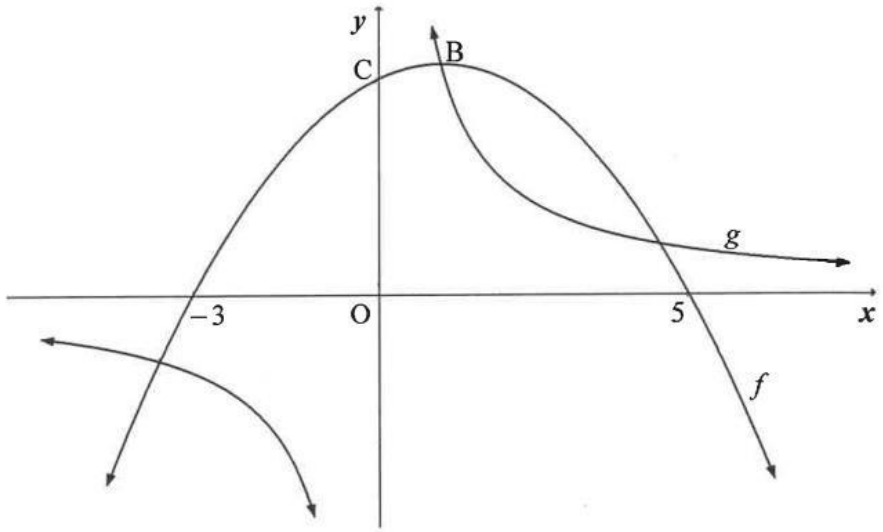
4.5 Write down the equation of g if it is given that $g(x) = f(x) + 4$ (1)

4.6 Write down the domain of g^{-1}. (2)

4.7 Write down the equation of g^{-1} in the form $y = \ldots$ (2)
[14]

QUESTION 5

The graphs of $f(x) = -\frac{1}{2}(x-1)^2 + 8$ and $g(x) = \frac{d}{x}$ are drawn below. A point of intersection of f and g is B, the turning point of f. The graph f has x-intercepts at $(-3\ ;\ 0)$ and $(5\ ;\ 0)$ and a y-intercept at C.



5.1 Write down the coordinates of the turning point of f. (2)

5.2 Calculate the coordinates of C. (2)

5.3 Calculate the value of d. (1)

5.4 Write down the range of g. (1)

5.5 For which values of x will $f(x).g(x) \leq 0$? (3)

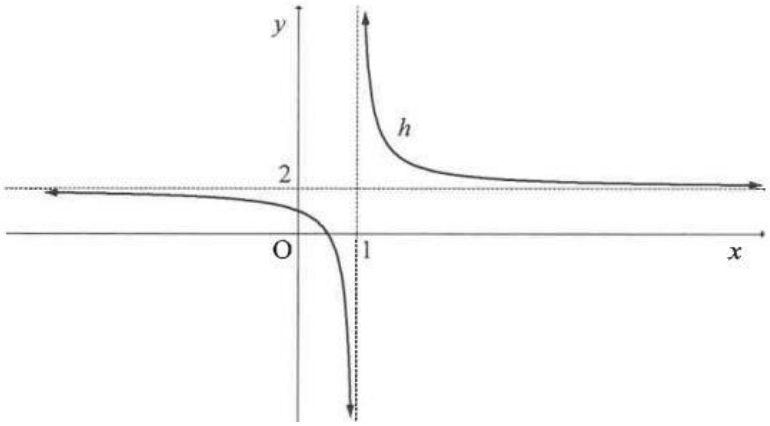
5.6 Calculate the values of k so that $h(x) = -2x + k$ will not intersect the graph of g. (5)

5.7 h is a tangent to g at R, a point in the first quadrant. Calculate t such that $y = f(x) + t$ intersects g at R. (4)
[18]

Paper C

QUESTION 4

4.1 Sketched below is the graph of $h(x)=\frac{1}{x+p}+q$. The asymptotes of h intersect at $(1\,;2)$.



4.1.1 Write down the values of p and q. (2)

4.1.2 Calculate the coordinates of the x-intercept of h. (2)

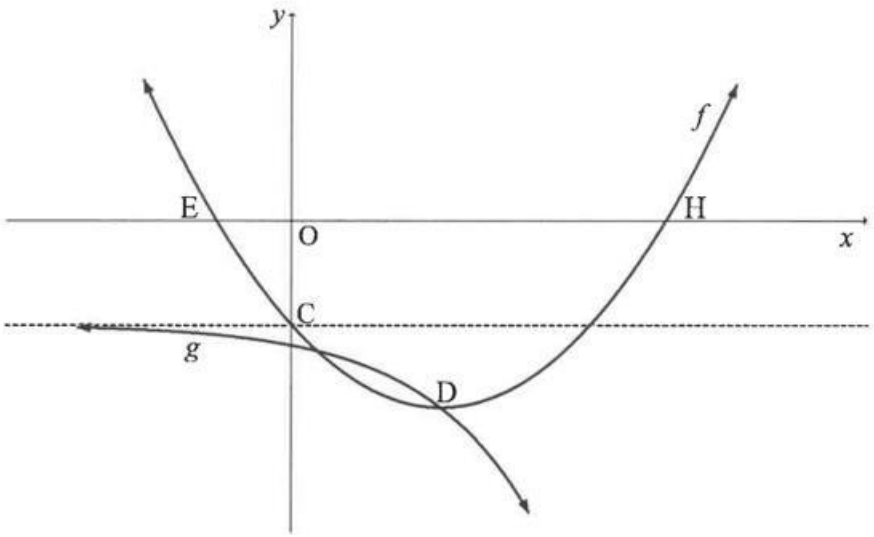
4.1.3 Write down the x-coordinate of the x-intercept of g if $g(x)=h(x+3)$. (2)

4.1.4 The equation of an axis of symmetry of h is $y=x+t$. Determine the value of t. (2)

4.1.5 Determine the values of x for which $-2\leq\frac{1}{x-1}$. (3)

4.2 The graphs of $f(x) = x^2 - 4x - 5$ and $g(x) = a.2^x + q$ are sketched below.

- E and H are the x-intercepts of f.
- C is the y-intercept of f and lies on the asymptote of g.
- The two graphs intersect at D, the turning point of f.



4.2.1 Write down the y-coordinate of C. (1)

4.2.2 Determine the coordinates of D. (2)

4.2.3 Determine the values of a and q. (3)

4.2.4 Write down the range of g. (1)

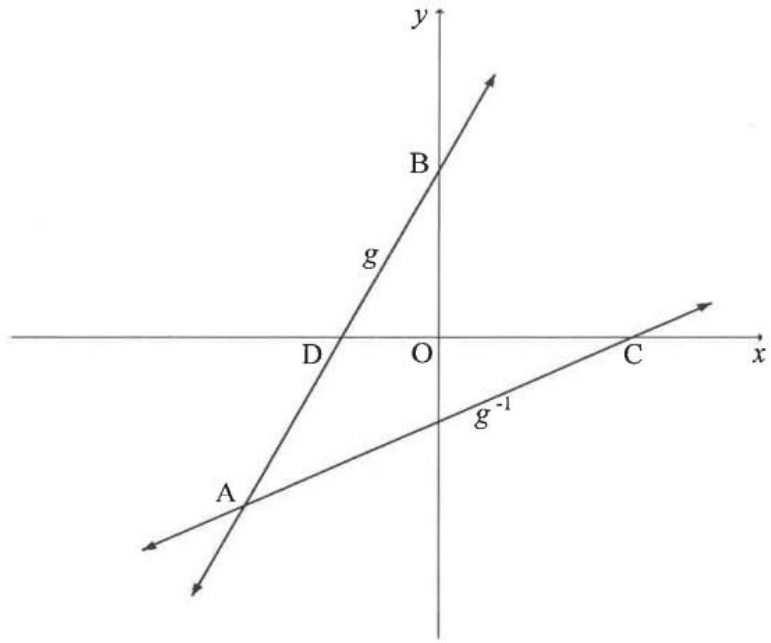
4.2.5 Determine the values of k for which the value of $f(x) - k$ will always be positive. (2)

[20]

QUESTION 5

The graphs of $g(x) = 2x + 6$ and g^{-1}, the inverse of g, are shown in the diagram below.

- D and B are the x- and y-intercepts respectively of g.
- C is the x-intercept of g^{-1}.
- The graphs of g and g^{-1} intersect at A.



5.1 Write down the y-coordinate of B. (1)

5.2 Determine the equation of g^{-1} in the form $g^{-1}(x) = mx + n$. (2)

5.3 Determine the coordinates of A. (3)

5.4 Calculate the length of AB. (2)

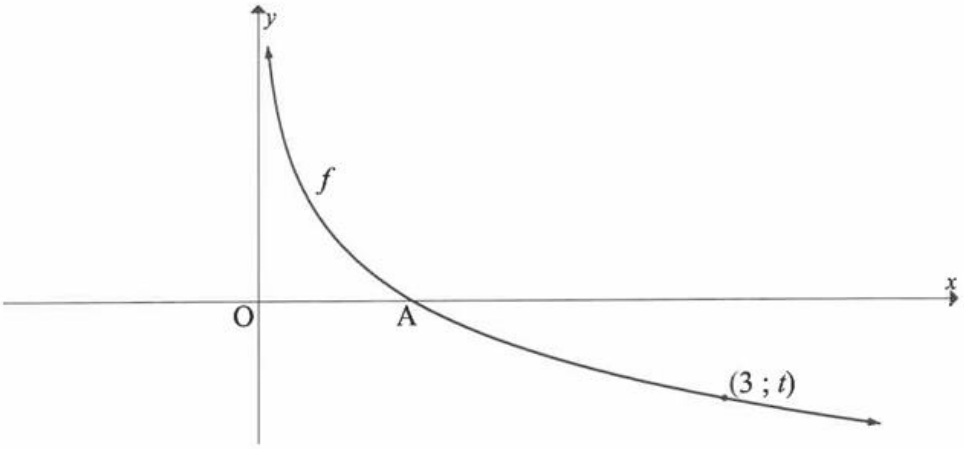
5.5 Calculate the area of ΔABC. (5)

[13]

Paper D

QUESTION 4

The graph of $f(x) = \log_{\frac{1}{3}} x$ is drawn below. Point A is the x-intercept of f and $(3\ ;\ t)$ lies on f.



4.1 Calculate the value of t (1)

4.2 Write down the coordinates of A. (1)

4.3 Determine the equation of f^{-1}, the inverse of f, in the form $y = \ldots$ (2)

4.4 Write down the equation of the asymptote of f^{-1} (1)

4.5 Draw the graph of f^{-1} on the set of axes provided in the ANSWER BOOK. Clearly indicate the intercepts with the axes, the coordinates of ONE other point and the asymptotes. (3)

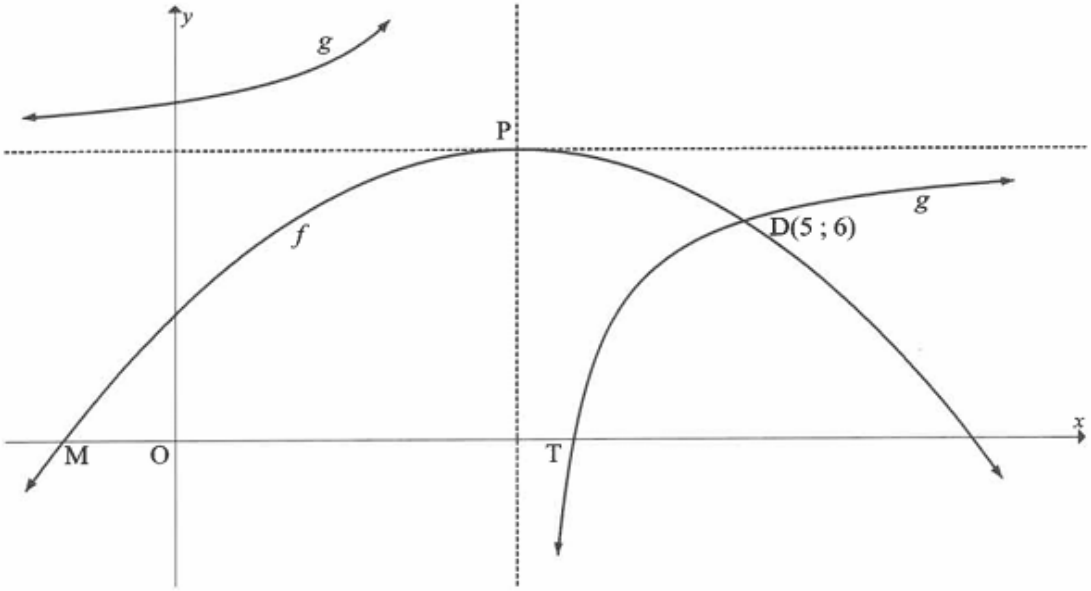
4.6 The graph of h is obtained when f^{-1} is translated 5 units to the right. Determine the y-values of h where $x > 4$. (2)

[10]

QUESTION 5

The graphs of $f(x)=ax^2+bx+c$ and $g(x)=\frac{-4}{x-3}+8$ are drawn below.

P is the turning point of f and the point of intersection of the asymptotes of g.
The graphs intersect at D(5 ; 6). M and T are x-intercepts of f and g respectively.



5.1 Write down the domain of g. (1)

5.2 Write down the range of f. (1)

5.3 Determine the values of x for which:

5.3.1 $g(x) \leq f(x)$ (2)

5.3.2 $f(x) < 6$ (2)

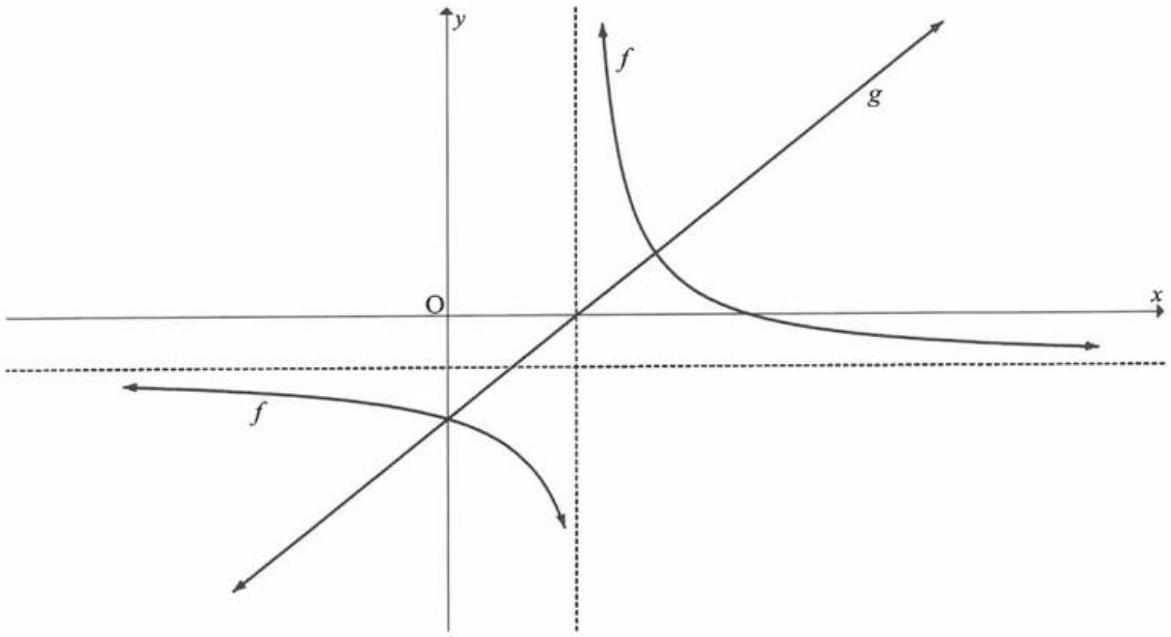
5.4 Show that the equation of the parabola is $f(x)=-\frac{1}{2}x^2+3x+\frac{7}{2}$ (3)

5.5 Calculate the length of MT. (6)

5.6 Determine the equation of the tangent to f at D. (3)
[18]

QUESTION 6

The graphs of $g(x) = x + c$ and $f(x) = \frac{a}{x+p} + q$ are drawn below. Graph g and the vertical asymptote of f intersect at the x-axis.



6.1 Write down the coordinates of the x-intercept of g in terms of p. (1)

6.2 Graph g intersects the horizontal asymptote of graph f at $x = 1$ and the graph f at $x = 3$. Graphs f and g also intersect on the y-axis. Determine the equation of f. (5)

6.3 Describe the transformation that g must undergo to become an axis of symmetry of f that cuts f at two points. (2)

[8]

Paper E

QUESTION 4

Given: $f(x) = a^x - 1$ for $a > 0$. B$\left(2; \frac{-5}{9}\right)$ is a point on f.

4.1 Calculate the value of a. (2)

4.2 Write down the range of f. (1)

4.3 Sketch the graph of f. Clearly show the intercepts with the axes and asymptotes, if any. (3)

4.4 It is further given that C is a point on f at $y = \frac{19}{8}$.

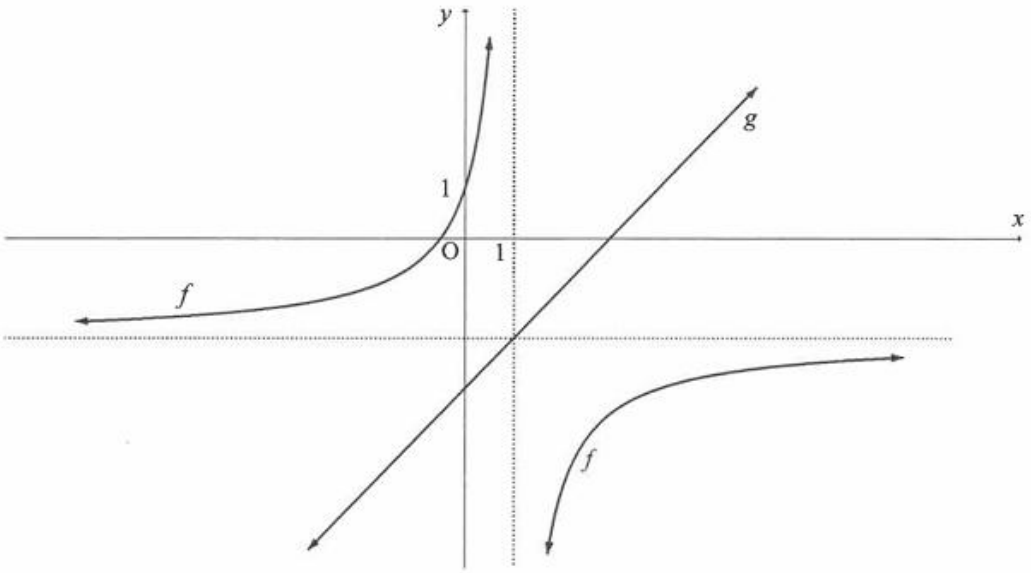
Determine the coordinates of C$'$, the image of C, when C is reflected about the line $y = x$. (3)

[9]

QUESTION 5

Sketched below is the graph of $f(x)=\frac{a}{x+p}+q$ having the domain $(-\infty;1)\cup(1;\infty)$.

The graph of f cuts the y-axis at $(0;1)$. A line of symmetry of f is given by $g(x)=x-3$.



5.1 Write down the value of p. (1)

5.2 Determine the equation of the horizontal asymptote of f. (2)

5.3 Calculate the value of a. (2)

5.4 For which values of x is $f(x)\geq 0$? (3)

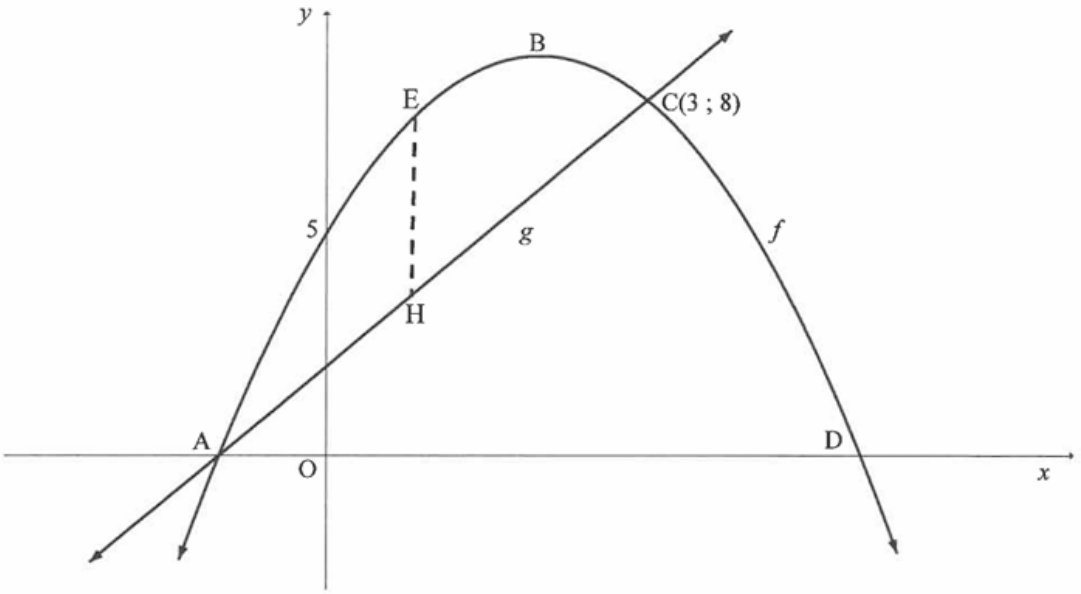
5.5 Graph f undergoes a transformation to h where:

- The domain and range of h are the same as that of f
- $h'(x)$, the derivative of h, is negative on its domain

Describe a possible transformation that f could have undergone to result in h. (2)
[10]

QUESTION 6

In the diagram below, the graphs of $f(x) = -x^2 + 4x + 5$ and g, a straight line, are drawn. $C(3\,;8)$ is a point of intersection of f and g. EH is drawn parallel to the y-axis, with E a point on f and H a point on g.



6.1 Calculate the coordinates of B, the turning point of f. (3)

6.2 Show that the equation of the line through A and C is given by $g(x) = 2x + 2$. (3)

6.3 Calculate the maximum length of EH for $f > g$. (4)

6.4 Given: $k(x) = f(x + m) = -x^2 - 2mx - m^2 + 4x + 4m + 5$

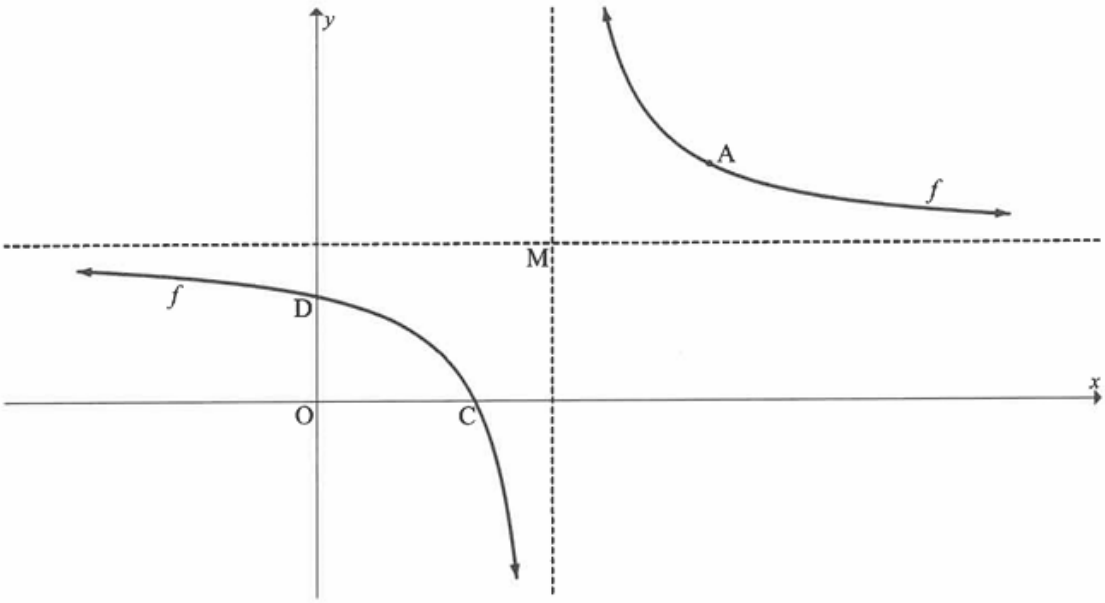
Determine the value of m such that g is a tangent to k. (5)

[15]

Paper F

QUESTION 4

The graph of $f(x)=\frac{4}{x-3}+4$ is drawn below. M is the point where the asymptotes of f intersect. C and D are the x- and y-intercepts respectively of f. A is the point on f that is closest to M.



4.1 Write down the coordinates of M. (2)

4.2 Calculate the coordinates of D. (2)

4.3 If $y=x+t$ is the equation of a line of symmetry of f, calculate the value of t. (2)

4.4 Determine the values of x for which $f(x)\leq 0$. (4)

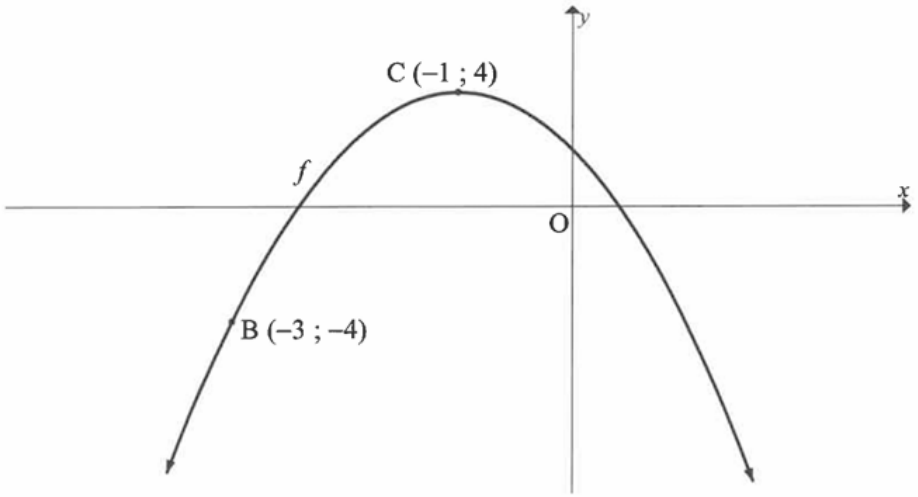
4.5 Calculate the coordinates of A. (3)

4.6 A single transformation is applied to f to obtain a new graph defined as $h(x)=\frac{-4}{x+3}+4$. A$'$ is the image of A under this transformation. Calculate the length of AA$'$. (2)

[15]

QUESTION 5

The graph of $f(x) = a(x+p)^2 + q$ is drawn below. C(– 1 ; 4) is the turning point of f.
B(– 3 ; – 4) is a point on f.



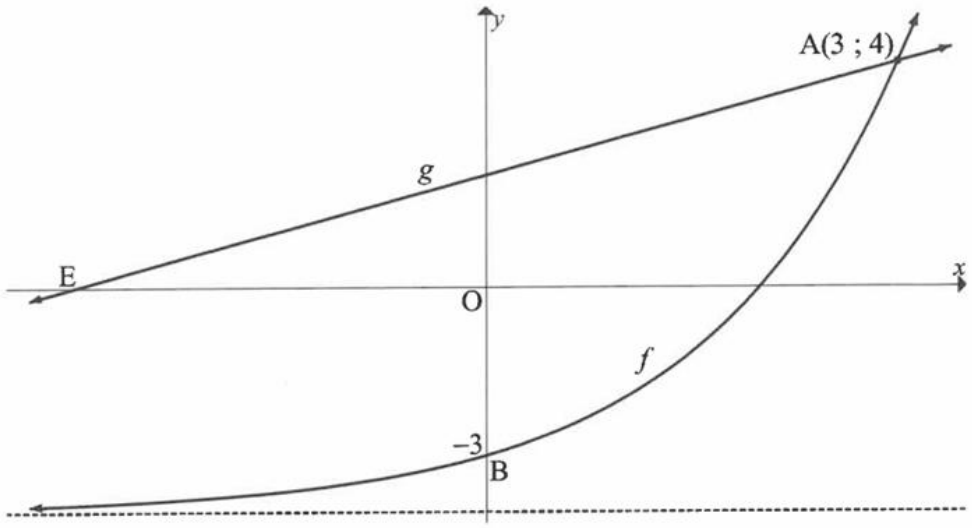
5.1 Show that $f(x) = -2x^2 - 4x + 2$. (3)

5.2 Determine the values of k for which $h(x) = f(x) + k$ will have no real roots. (2)

5.3 The graph of $y = g'(x)$, where g' is the derivative of g, is obtained when f is reflected over the line $y = 4$. Draw a sketch graph of g if $g(0) < 0$. Clearly indicate any stationary points on your graph. (4)
[9]

QUESTION 6

The graphs of $f(x) = p^x + q$ and $g(x) = mx + c$ are drawn below. A(3 ; 4) is the point of intersection of f and g. B(0 ; – 3) is the y-intercept of f. E is the x-intercept of g.



6.1 Calculate the values of p and q. (4)

6.2 Write down the range of f. (1)

6.3 The graph of g^{-1}, the inverse of g, also passes through B. Determine the equation of g in the form $y = \ldots$ (4)

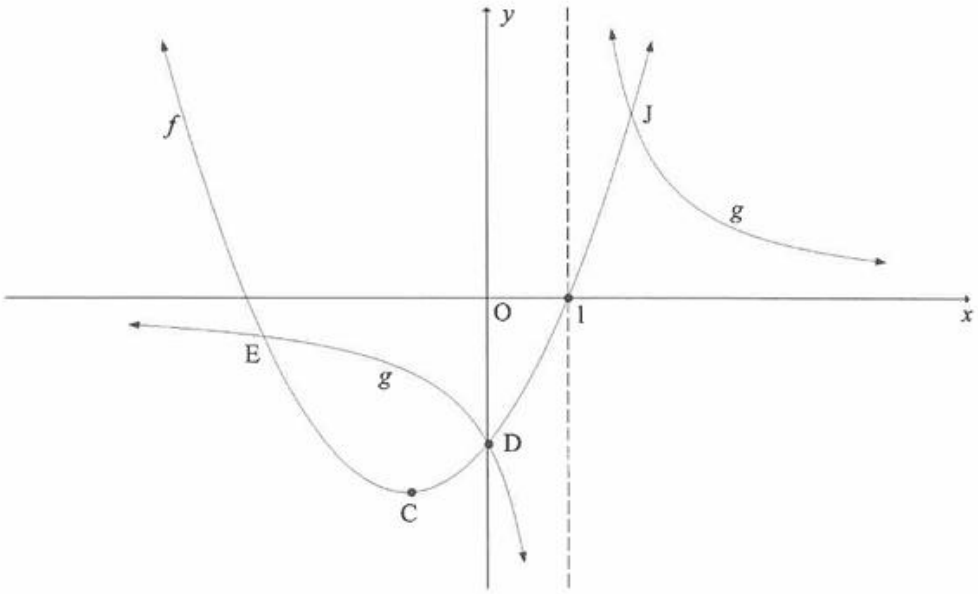
6.4 Write down the equation of g^{-1} in the form $y = \ldots$ (2)
[11]

Paper G

QUESTION 4

Below are the graphs of $f(x) = x^2 + bx - 3$ and $g(x) = \frac{a}{x+p}$.

- f has a turning point at C and passes through the x-axis at $(1\ ;\ 0)$.
- D is the y-intercept of both f and g. The graphs f and g also intersect each other at E and J.
- The vertical asymptote of g passes through the x-intercept of f.



4.1 Write down the value of p. (1)

4.2 Show that $a = 3$ and $b = 2$. (3)

4.3 Calculate the coordinates of C. (4)

4.4 Write down the range of f. (2)

4.5 Determine the equation of the line through C that makes an angle of $45°$ with the positive x-axis. Write your answer in the form $y = \ldots$ (3)

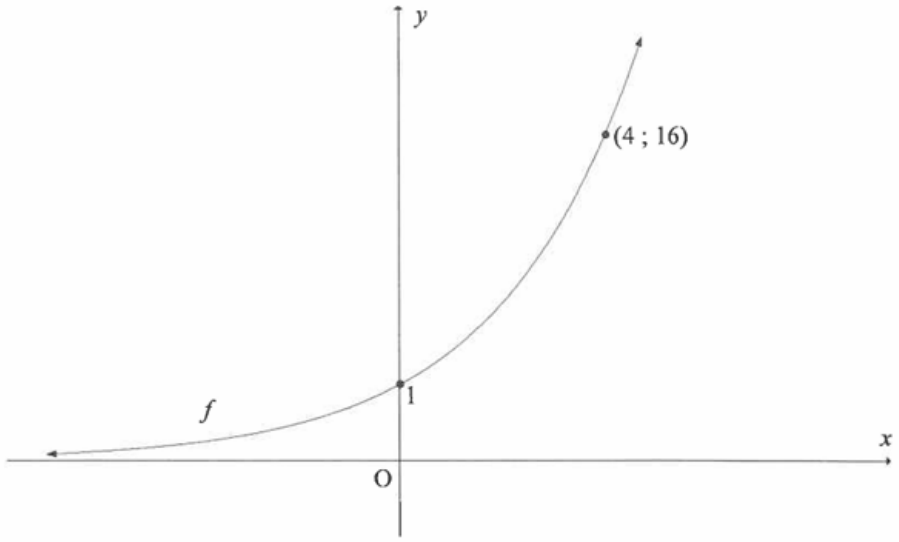
4.6 Is the straight line, determined in QUESTION 4.5, a tangent to f? Explain your answer. (2)

4.7 The function $h(x) = f(m - x) + q$ has only one x-intercept at $x = 0$. Determine the values of m and q. (4)

[19]

QUESTION 5

Sketched below is the graph of $f(x) = k^x$; $k > 0$. The point $(4 ; 16)$ lies on f.



5.1 Determine the value of k. (2)

5.2 Graph g is obtained by reflecting graph f about the line $y = x$. Determine the equation of g in the form $y = \ldots$ (2)

5.3 Sketch the graph g. Indicate on your graph the coordinates of two points on g. (4)

5.4 Use your graph to determine the value(s) of x for which:

5.4.1 $f(x) \times g(x) > 0$ (2)

5.4.2 $g(x) \leq -1$ (2)

5.5 If $h(x) = f(-x)$, calculate the value of x for which $f(x) - h(x) = \frac{15}{4}$ (4)

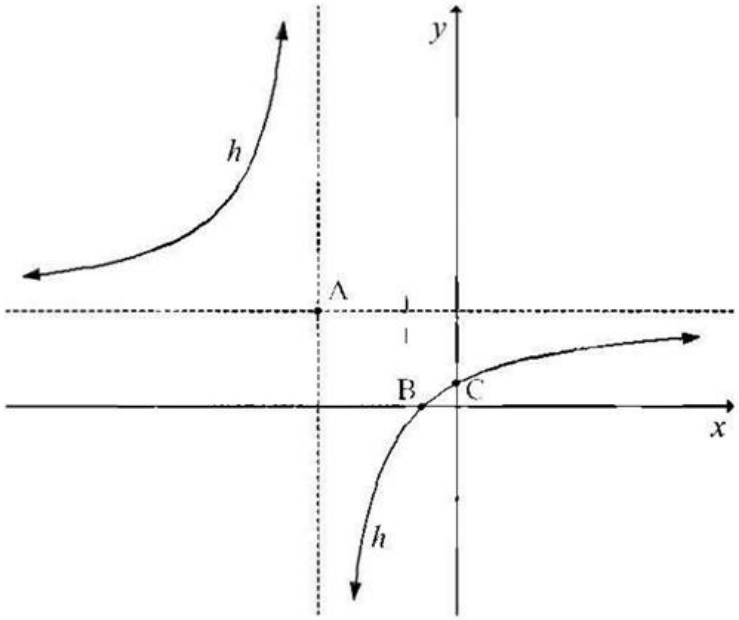
[16]

Paper H

QUESTION 4

The sketch below shows the graph of $h(x)=\frac{-9}{x+4}+3$. The asymptotes of h intersect at A.

The graph h intersects the x-axis and y-axis at B and C respectively.



4.1 Write down the coordinates of A. (1)

4.2 Calculate the coordinates of B. (2)

4.3 Calculate the coordinates of C. (2)

4.4 Describe the translation from h to $j(x)=\frac{-9}{x}$. (2)

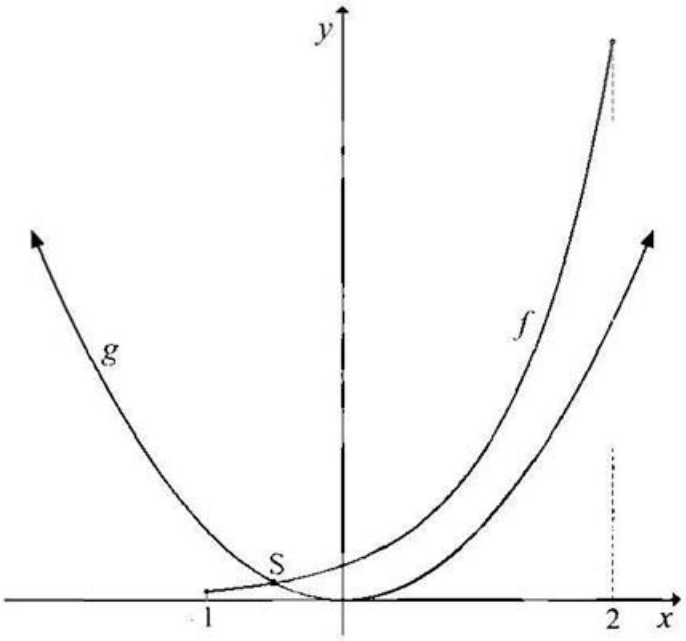
4.5 Determine the coordinates of the points on j that are closest to the origin. (4)

[11]

QUESTION 5

The diagram below shows the graphs of $f(x)=a^x$, for $x\in[-1\,;\,2]$, and $g(x)=bx^2$.

$S\left(-\frac{1}{2}\,;\,\frac{1}{2}\right)$ is a point of intersection of f and g.



5.1 Determine the values of a and b. (4)

5.2 Draw a sketch graph of the inverse of g. Indicate the coordinates of one point on the graph. (2)

5.3 Is the inverse of g a function? (1)

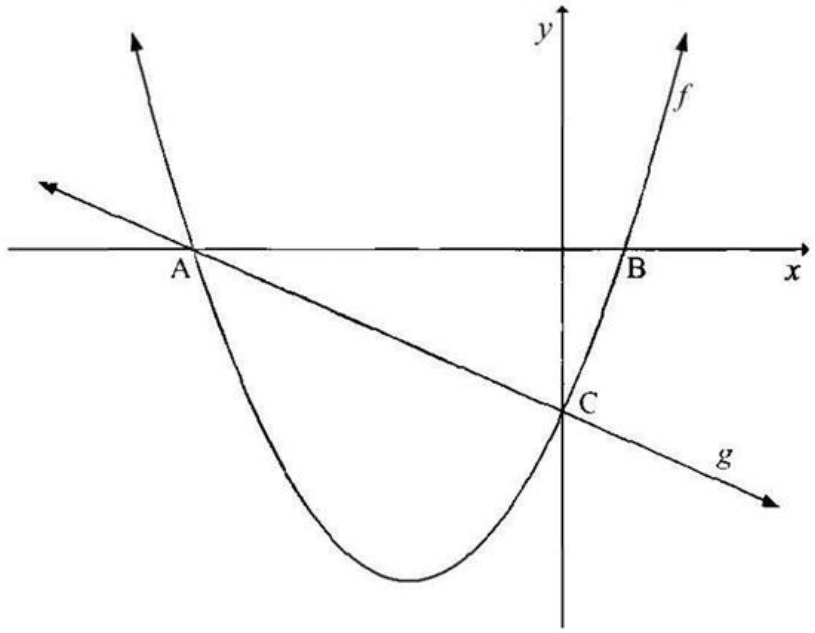
5.4 Determine the equation of f^{-1} in the form of $y=.....$
Also state the restriction on the domain. (4)

5.5 If $x<0$, write down the values of x for which $f(x)>g(x)$. (2)

[13]

QUESTION 6

The graphs of $f(x)=x^2+5x-6$ and $g(x)=mx+c$ are drawn below.
A and B are the x-intercepts of f and C, the y-intercept. g passes through A and C.



6.1 Calculate the coordinates of A and B. (3)

6.2 Determine the equation of g. (3)

6.3 If $h(x)=f(x)+k$, determine the values of k for which g and h will not intersect. (5)

[11]

Paper I

QUESTION 4

4.1 Given the functions $f(x)=x^2-2x-3$ and $g(x)=x-3$, where $x \in \mathbb{R}$.

4.1.1 Sketch the graphs of the two functions $f(x)$ and $g(x)$ on the same system of axes. Clearly indicate the coordinates of the turning point of $f(x)$, the axis of symmetry of $f(x)$ and the x- and y-intercepts of both graphs. (6)

4.1.2 Determine the value(s) of x, for which:

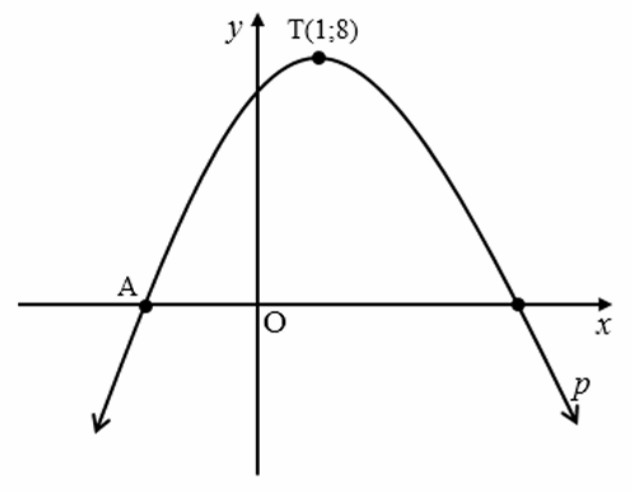
(a) $f(x)>g(x)$ (2)

(b) $f(x).g(x)\leq 0$ (2)

4.1.3 Determine the range of h, if $h(x)=-[f(x)+4]$. (2)

4.2 The following information of a parabola, p sketched below, is given:

- Coordinates of the turning point are T(1;8)
- $p'(x) = -x + 1$
- Gradient of the tangent at point A, an x-intercept of p, is 4

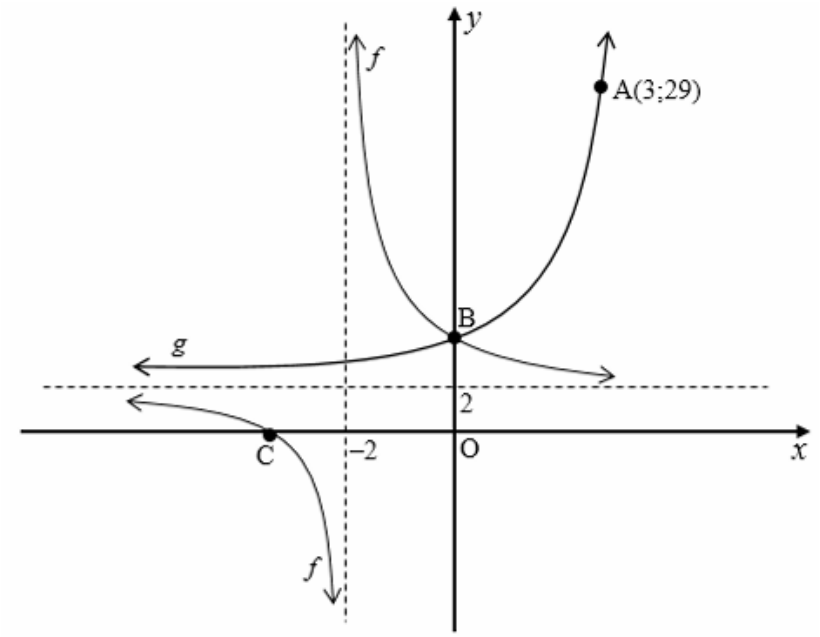


Determine the equation of the parabola, in the form $y = \ldots$ (4)

[16]

QUESTION 5

In the diagram below, the graphs of $f(x)=\frac{2}{x+p}+q$ and $g(x)=b^x+2$ are given, with point $A(3\,;\,29)$ on g.



5.1 Write down the coordinates of B. (1)

5.2 Write down the range of $g(x)$. (1)

5.3 Determine the value of b. (2)

5.4 Determine the coordinates of C, the x-intercept of f. (2)

5.5 If $h(x)=g(x)-2$, determine the equation of $h^{-1}(x)$, in the form $y=$ ... (3)

5.6 Determine the equation of the axis of symmetry of f, which has a positive gradient. (2)

5.7 For which values of x is $f(x)\times g'(x)\leq 0$? (2)

5.8 Determine the area of ΔABC. (4)

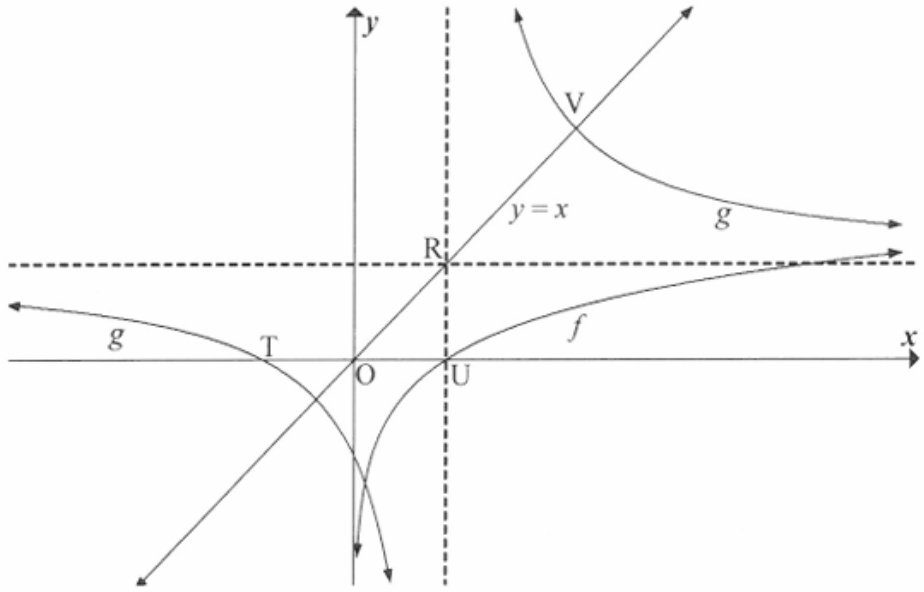
[17]

Paper J

QUESTION 4

The sketch below shows the graphs of $f(x) = \log_5 x$ and $g(x) = \frac{2}{x-1} + 1$.

- T and U are the x-intercepts of g and f respectively.
- The line $y = x$ intersects the asymptotes of g at R, and the graph of g at V.



4.1 Write down the coordinates of U. (1)

4.2 Write down the equations of the asymptotes of g. (2)

4.3 Determine the coordinates of T. (2)

4.4 Write down the equation of h, the reflection of f in the line $y = x$, in the form $y = ...$ (2)

4.5 Write down the equation of the asymptote of $h(x-3)$. (1)

4.6 Calculate the coordinates of V. (4)

4.7 Determine the coordinates of T′ the point which is symmetrical to T about the point R. (2)

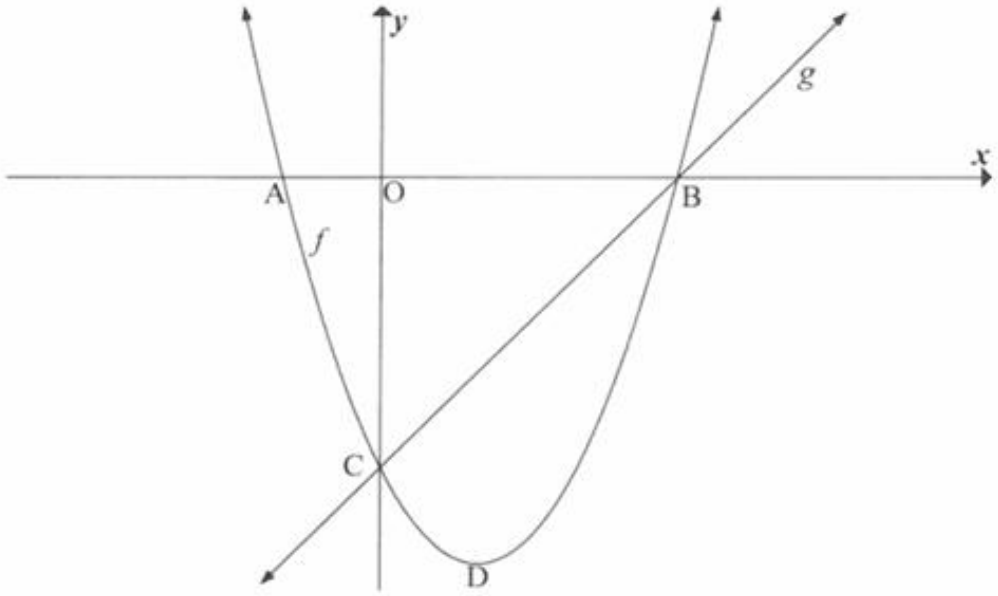
[14]

QUESTION 5

5.1 The sketch below shows the graphs of $f(x) = x^2 - 2x - 3$ and $g(x) = x - 3$.

- A and B are the x-intercepts of f.
- The graphs of f and g intersect at C and B.

D is the turning point of f.



5.1.1 Determine the coordinates of C. (1)

5.1.2 Calculate the length of AB. (4)

5.1.3 Determine the coordinates of D. (2)

5.1.4 Calculate the average gradient of f between C and D. (2)

5.1.5 Calculate the size of $O\hat{C}B$ (2)

5.1.6 Determine the values of k for which $f(x) = k$ will have two unequal positive real roots. (3)

5.1.7 For which values of x will $f'(x) \cdot f''(x) > 0$? (3)

5.2 The graph of a parabola f has x-intercepts at $x = 1$ and $x = 5$. $g(x) = 4$ is a tangent to f at P, the turning point of f. Sketch the graph of f, clearly showing the intercepts with the axes and the coordinates of the turning point. (5)

[22]

FINANCE, GROWTH AND DECAY

Paper A

QUESTION 7

7.1 A travel company informs Robert that a holiday to a certain destination costs R40 000 now. The company predicts that the cost of this holiday will increase by 7,8% per annum. What will this holiday cost in 5 years' time? (2)

7.2 Sarah opened a savings account that paid interest at a rate of 5,8% p.a., compounded quarterly. She deposited R2 300 into the account on 1 January 2020 and continued to make deposits of R2 300 at the beginning of each quarter thereafter. She made her last deposit on 1 October 2025. Calculate the accumulated amount in the account on 1 January 2026. (4)

7.3 The bank granted Rajesh a loan of R900 000 on 28 February 2024 at an interest rate of 6,8% p.a., compounded monthly.

7.3.1 Rajesh was unable to make the first three payments. He made his first repayment of R10 000 on 30 June 2024. He continued to make monthly repayments of R10 000 at the end of each month thereafter. How long, in completed months, will it take Rajesh to repay the loan from the time the loan was granted? (5)

7.3.2 Calculate the value of the final payment. (4)

[15]

Paper B

QUESTION 6

6.1 Patrick deposited an amount of R18 500 into an account earning r% interest p.a., compounded monthly. After 6 months, his balance was R19 319,48.

6.1.1 Calculate the value of r. (3)

6.1.2 Calculate the effective interest rate. (2)

6.2 Kuda bought a laptop for R10 000 on 31 January 2019. He will replace it with a new one in 5 years' time on 31 January 2024.

6.2.1 The value of the old laptop depreciates annually at a rate of 20% p.a. according to the straight-line method. After how many years will the laptop have a value of R0? (2)

6.2.2 Kuda will buy a laptop that costs R20 000. In order to cover the cost price, he made his first monthly deposit into a savings account on 28 February 2019. He will make his 60th monthly deposit on 31 January 2024. The savings account pays interest at 8,7% p.a., compounded monthly. Calculate Kuda's monthly deposit into this account. (4)

6.3 Tino wins a jackpot of R1 600 000. He invests all of his winnings in a fund that earns interest of 11,2% p.a., compounded monthly. He withdraws R20 000 from the fund at the end of each month. His first withdrawal is exactly 1 month after his initial investment. How many withdrawals of R20 000 will Tino be able to make from this fund? (5)

[16]

Paper C

QUESTION 6

6.1 R12 000 was invested in a fund that paid interest at m% p.a., compounded quarterly. After 24 months, the value of the investment was R13 459.

Determine the value of m. (4)

6.2 On 31 January 2022, Tino deposited R1 000 in an account that paid interest at 7,5% p.a., compounded monthly. He continued depositing R1 000 on the last day of every month. He will make the last deposit on 31 December 2022.

Will Tino have sufficient funds in the account on 1 January 2023 to buy a computer that costs R13 000? Justify your answer by means of an appropriate calculation. (4)

6.3 Thabo plans to buy a car that costs R250 000. He will pay a deposit of 15% and take out a loan for the balance. The interest on the loan is 13% p.a., compounded monthly.

6.3.1 Calculate the value of the loan. (1)

6.3.2 The first repayment will be made 6 months after the loan has been granted. The loan will be repaid over a period of 6 years after it has been granted. Calculate the MONTHLY instalment. (5)

[14]

Paper D

QUESTION 7

7.1 John invests an amount of money in an account paying interest at a rate of 15% p.a., compounded monthly. Calculate the annual effective interest rate of this investment. (2)

7.2 Tino invests R500 000 in an account earning interest at the rate of 6% p.a., compounded quarterly. Tino decides to withdraw R11 250 at the end of every 3 months. Tino will continue making these regular withdrawals until there is no money in the account. How many withdrawals of R11 250 will Tino be able to make? (5)

7.3 On **1 March 2021**, Abby made a once-off deposit of R12 000 into an account earning interest at a rate of 9,5% p.a., compounded monthly. She deposited R500 into the same account on **1 April 2023** and continues to make these monthly deposits of R500 on the first day of each month thereafter.

Calculate how much money was in the account immediately after the deposit of R500 is made on **1 March 2025**, exactly four full years after her initial deposit. (6)
[13]

Paper E

QUESTION 7

7.1 Mary's grandparents deposited R5 000 into a savings account on the day that she was born. The account pays interest at a rate of 6,8% p.a., compounded quarterly. Calculate the accumulated amount in this account on Mary's 16th birthday. (3)

7.2 After 4 years, the value of a printer was half of its original value. Determine the rate at which the value of the printer depreciated over this period, if depreciation was calculated according to a straight-line method. (2)

7.3 Tshepo was granted a loan of R100 000 on 1 March 2022 at an interest rate of 13,5% p.a., compounded monthly. Tshepo agreed to repay the loan over 5 years in monthly instalments of R2 300,98, starting on 1 April 2022.

7.3.1 Calculate the total interest that he will pay over the 5 years. (2)

7.3.2 Tshepo paid R22 300,98 (his monthly instalment and an additional R20 000) on 1 March 2024 into the loan account. He continues to pay the original monthly instalment thereafter. How many months earlier will Tshepo repay the loan? (7)
[14]

Paper F

QUESTION 6

6.1 On the 2nd day of January 2015 a company bought a new printer for R150 000.

- The value of the printer decreases by 20% annually on the reducing-balance method.
- When the book value of the printer is R49 152, the company will replace the printer.

6.1.1 Calculate the book value of the printer on the 2nd day of January 2017. (3)

6.1.2 At the beginning of which year will the company have to replace the printer? Show ALL calculations. (4)

6.1.3 The cost of a similar printer will be R280 000 at the beginning of 2020. The company will use the R49 152 that it will receive from the sale of the old printer to cover some of the costs of replacing the printer. The company set up a sinking fund to cover the balance. The fund pays interest at 8,5% per annum, compounded quarterly. The first deposit was made on 2 April 2015 and every three months thereafter until 2 January 2020. Calculate the amount that should be deposited every three months to have enough money to replace the printer on 2 January 2020. (4)

6.2 Lerato wishes to apply for a home loan. The bank charges interest at 11% per annum, compounded monthly. She can afford a monthly instalment of R9 000 and wants to repay the loan over a period of 15 years. She will make the first monthly repayment one month after the loan is granted. Calculate, to the nearest thousand rand, the maximum amount that Lerato can borrow from the bank. (5)

[16]

Paper G

QUESTION 6

6.1 R12 000 was invested in a fund that paid interest at m% p.a., compounded quarterly. After 24 months, the value of the investment was R13 459.

Determine the value of m. (4)

6.2 On 31 January 2022, Tino deposited R1 000 in an account that paid interest at 7,5% p.a., compounded monthly. He continued depositing R1 000 on the last day of every month. He will make the last deposit on 31 December 2022.

Will Tino have sufficient funds in the account on 1 January 2023 to buy a computer that costs R13 000? Justify your answer by means of an appropriate calculation. (4)

6.3 Thabo plans to buy a car that costs R250 000. He will pay a deposit of 15% and take out a loan for the balance. The interest on the loan is 13% p.a., compounded monthly.

6.3.1 Calculate the value of the loan. (1)

6.3.2 The first repayment will be made 6 months after the loan has been granted. The loan will be repaid over a period of 6 years after it has been granted. Calculate the MONTHLY instalment. (5)

[14]

Paper H

QUESTION 6

6.1 Two friends, Kuda and Thabo, each want to invest R5 000 for four years. Kuda invests his money in an account that pays simple interest at 8,3% per annum. At the end of four years, he will receive a bonus of exactly 4% of the accumulated amount. Thabo invests his money in an account that pays interest at 8,1% p.a., compounded monthly.

Whose investment will yield a better return at the end of four years? Justify your answer with appropriate calculations. (5)

6.2 Nine years ago, a bank granted Mandy a home loan of R525 000. This loan was to be repaid over 20 years at an interest rate of 10% p.a., compounded monthly. Mandy's monthly repayments commenced exactly one month after the loan was granted.

6.2.1 Mandy decided to make monthly repayments of R6 000 instead of the required R5 066,36. How many payments will she make to settle the loan? (5)

6.2.2 After making monthly repayments of R6 000 for nine years, Mandy required money to fund her daughter's university fees. She approached the bank for another loan. Instead, the bank advised Mandy that the extra amount repaid every month could be regarded as an investment and that she could withdraw this full amount to fund her daughter's studies. Calculate the maximum amount that Mandy may withdraw from the loan account. (4)

[14]

Paper I

QUESTION 7

7.1 At what annual percentage interest rate, compounded quarterly, should a lump sum be invested in order for it to double in 6 years? (3)

7.2 Micaela buys furniture to the value of R10 000. She borrows the money on 1 February 2023 from a financial institution that charges interest at a rate of 9,5% *p.a.* compounded monthly. Micaela agrees to pay monthly instalments of R450. The loan agreement allows Micaela to start paying equal monthly instalments from 01 August 2023.

7.2.1 Calculate the total amount owing to the financial institution on 1 July 2023. (3)

7.2.2 How many months will it take Micaela to pay back the loan? (4)

7.2.3 What is the balance of the loan immediately after Micaela has made the 25th payment? (3)

[13]

Paper J

QUESTION 6

6.1 How long must R50 000 be invested to double at an interest rate of 8.5% p.a. on the straight-line method? (Give your answer in years and months). (3)

6.2 On 1 June 2024, a bank granted Thabiso a loan of R250 000 at an interest rate of 15% p.a. compounded monthly to buy a car. Thabiso agreed to repay the loan in monthly instalments commencing on 1 July 2024 and ending four (4) years later on 1 June 2028. However, Thabiso was unable to make the first two instalments and only commenced with monthly instalments on 1 September 2024.

6.2.1 Calculate the amount Thabiso owed the bank on 1 August 2024, a month before he paid his first monthly instalment. (2)

6.2.2 Having paid his first monthly instalment on 1 September 2024, Thabiso will still pay his last monthly instalment on 1 June 2028. Calculate his monthly instalment. (4)

6.3 Rx (x Rands) is invested into a new account at an interest rate of 12% p.a. compounded monthly. Three years later, R$2x$ is deposited into the same account. After seven years, there is R276 558,75 in the account. Determine how much money was invested at the beginning. (That is the value of x.) (6)

[15]

DIFFERENTIAL CALCULUS

Paper A

QUESTION 8

8.1 Determine $f'(x)$ from first principles if $f(x)=\frac{1}{x}$. (5)

8.2 Determine:

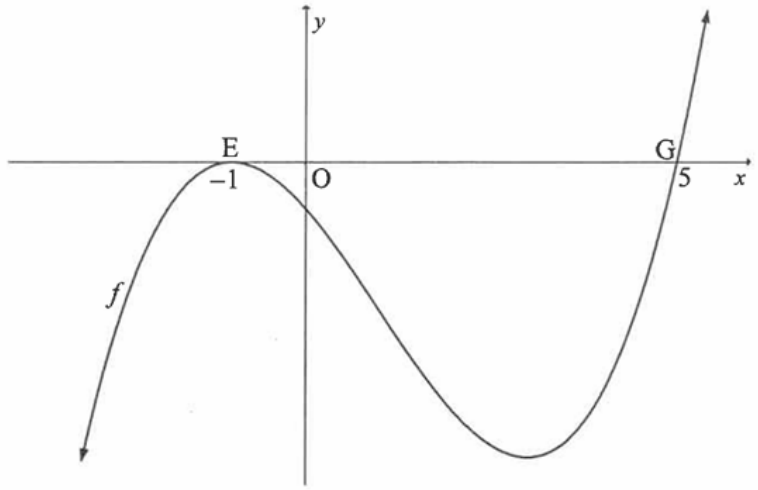
8.2.1 $\frac{d}{dx}\left(\sqrt{4x^6}+\sqrt{2}.x^2\right)$ (3)

8.2.2 $g'(x)$ if $g(x)=\frac{3x^4-4x^2+6}{x^2}$ (3)

8.3 The equation of the tangent to $f(x)=3x^2+bx+c$ at $x=1$ is given by $y=9x-9$.
Determine the values of b and c. (4)
[15]

QUESTION 9

The graph of $f(x) = ax^3 + bx^2 + cx - 5$ is drawn below. E(– 1 ; 0) and G(5 ; 0) are the x-intercepts of f.



9.1 Show that $a = 1$, $b = -3$ and $c = -9$. (3)

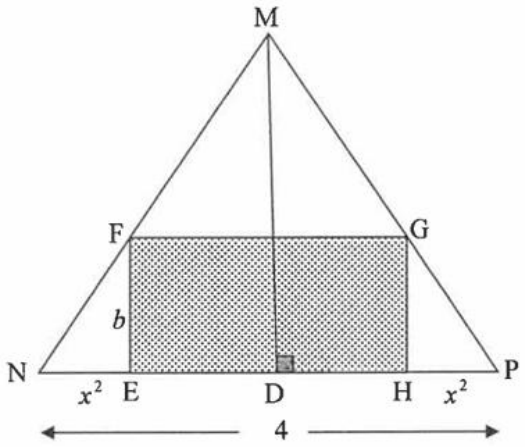
9.2 Calculate the value of x for which f has a local minimum value. (4)

9.3 Use the graph to determine the values of x for which $f''(x).f(x) > 0$. (3)

9.4 For which values of t will the graph of $p(x) = f(x) + t$ have two distinct positive roots and one negative root? (3)
[13]

QUESTION 10

EHGF is a rectangle. HE is produced x^2 cm to N and EH is produced x^2 cm to P. NF produced intersects PG produced at M to form an isosceles triangle MNP with NM = MP. D lies on NP where MD ⊥ NP. NP = 4 cm and MD = 3 cm.



10.1 Show that the area of EFGH is given by $A(x) = 6x^2 - 3x^4$. (4)

10.2 Calculate the maximum area of rectangle EFGH. (4)
[8]

Paper B

QUESTION 7

7.1 Determine $f'(x)$ from first principles if $f(x) = -4x^2$ (5)

7.2 Determine:

7.2.1 $f'(x)$ if $f(x) = 2x^3 - 3x$ (2)

7.2.2 $D_x\left(7\sqrt[3]{x^2} + 2x^{-5}\right)$ (3)

7.3 For which values of x will the tangent to $f(x) = -2x^3 + 8x$ have a positive gradient? (3)
[13]

QUESTION 8

Given: $f(x) = -x^3 + 6x^2 - 9x + 4 = (x-1)^2(-x+4)$

8.1 Determine the coordinates of the turning points of f. (4)

8.2 Draw a sketch graph of f. Clearly label all the intercepts with the axes and any turning points. (4)

8.3 Use the graph to determine the value(s) of k for which $-x^3 + 6x^2 - 9x + 4 = k$ will have three real and unequal roots. (2)

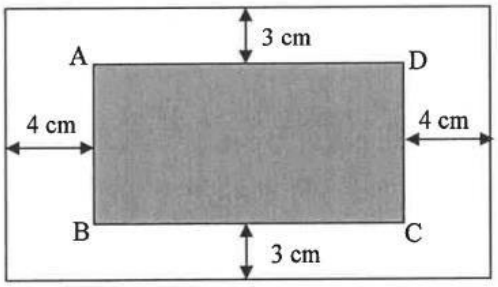
8.4 The line $g(x) = ax + b$ is the tangent to f at the point of inflection of f. Determine the equation of g. (6)

8.5 Calculate the value of θ, the acute angle formed between g and the x-axis in the first quadrant. (2)

[18]

QUESTION 9

The diagram below represents a printed poster. Rectangle ABCD is the part on which the text is printed. This shaded area ABCD is $432\ cm^2$ and $AD = x$ cm. ABCD is 4 cm from the left and right edges of the page and 3 cm from the top and bottom of the page.



9.1 Show that the total area of the page is given by:

$$A(x) = \frac{3\,456}{x} + 6x + 480$$ (3)

9.2 Determine the value of x such that the total area of the page is a minimum. (3)

[6]

Paper C

QUESTION 8

8.1 Determine $f'(x)$ from first principles if it is given that $f(x) = -2x + 3$. (4)

8.2 Determine:

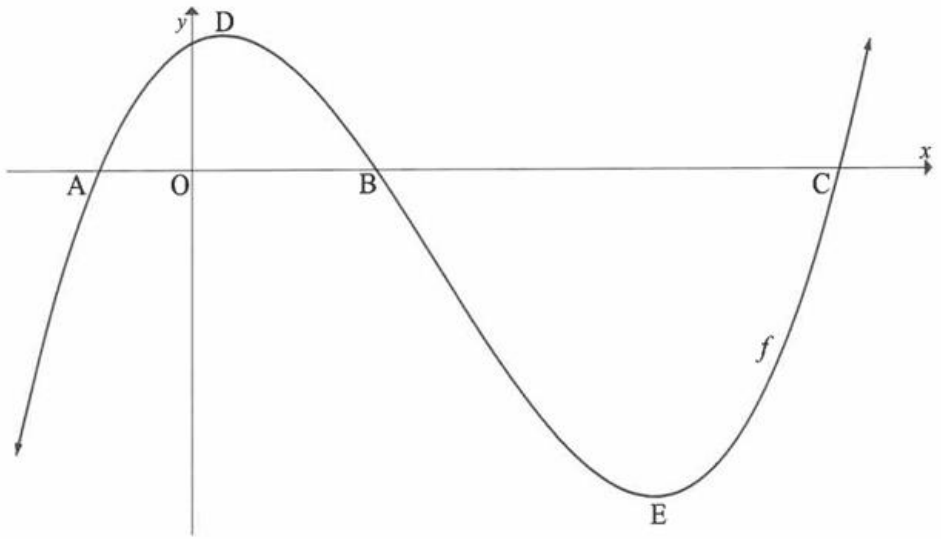
8.2.1 $g'(x)$ if $g(x) = -3x^4 + 2x$ (2)

8.2.2 $\frac{dy}{dx}$ if $y = \frac{2x^4 + 1}{x^2}$ (4)

[10]

QUESTION 9

Sketched below is the graph of $f(x) = x^3 - 8x^2 + 5x + 14$. A, B and C are the x–intercepts of f. D and E are the turning points of f.



9.1 Calculate the coordinates of E. (4)

9.2 For which values of x is f concave down? (3)

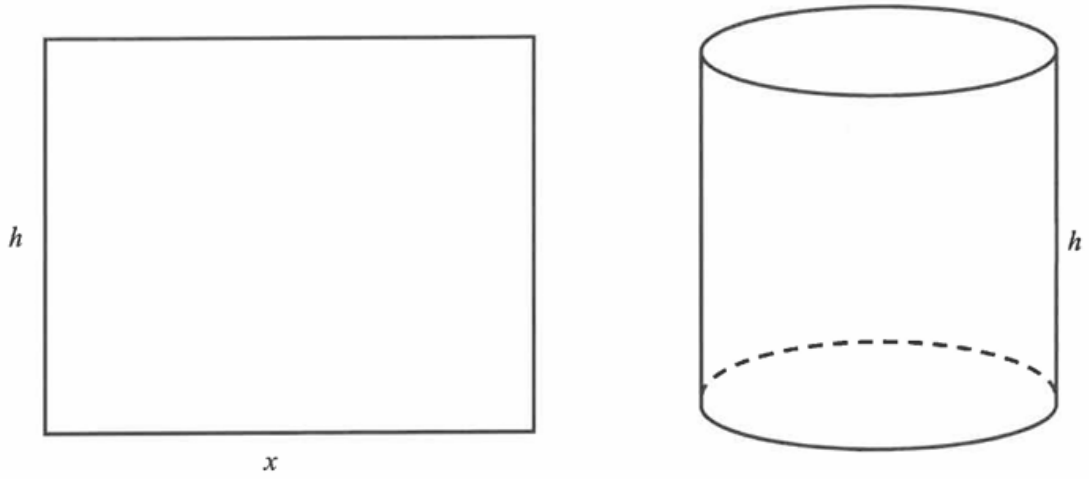
9.3 The coordinates of B are $(2\,;0)$. Use the graph to determine the values of x for which $f(x).f''(x) < 0$ (4)

9.4 For which values of t will $y = -11x + t$ intersect f at 3 distinct points? (6)

[17]

QUESTION 10

A rectangular metal sheet has dimensions x and h units, with $x > h$, and a perimeter of 50 units. The metal sheet is rolled into a cylinder with two open ends (top and bottom) and height h units.



10.1 Show that the volume of the cylinder is given by $V = \frac{25x^2}{4\pi} - \frac{x^3}{4\pi}$ (3)

10.2 Calculate the value of x that will maximise the volume of the cylinder. (3)
[6]

Paper D

QUESTION 8

8.1 Determine $f'(x)$ from first principles if it is given that $f(x) = x^2 - 2$ (5)

8.2 Determine:

8.2.1 $\frac{d}{dx}\left[3x^2 - 4x\right]$ (2)

8.2.2 $g'(x)$ if $g(x) = -2\sqrt{x}(x-1)^2$ (4)

8.3 Given that $y = 4x - 14$ is a common tangent to $f(x) = 2x^2 - 4x - 6$ and $g(x) = ax^2 + bx - 18$, calculate the values of a and b. (6)
[17]

QUESTION 9

Given: $f(x) = x^3 - 6x^2 + 9x - 4 = (x-4)(x-k)^2$

9.1 Show that $k = 1$. (2)

9.2 Calculate the coordinates of the turning points of f. (4)

9.3 Describe the concavity of f at $x = -3$. (2)

9.4 Draw the graph of f. Label ALL turning points and intercepts with the axes. (4)

9.5 Calculate the maximum vertical distance between f and h for $1 < x < 3$, if $h(x) = -2f'(x)$. (6)
[18]

Paper E

QUESTION 8

8.1 Determine:

8.1.1 $\frac{d}{dx}\left[3x - 5x^2\right]$ (2)

8.1.2 $g'(x)$ if $g(x) = \frac{2}{x^2} - \sqrt[3]{x^7}$ (4)

8.2 Determine the equation of the tangent to $f(x) = x^3 - 4x^2 + 2x + 3$ at $x = 2$. (3)

8.3 Given: $f(x) = -6x^2$

8.3.1 Determine $f'(x)$ from first principles. (5)

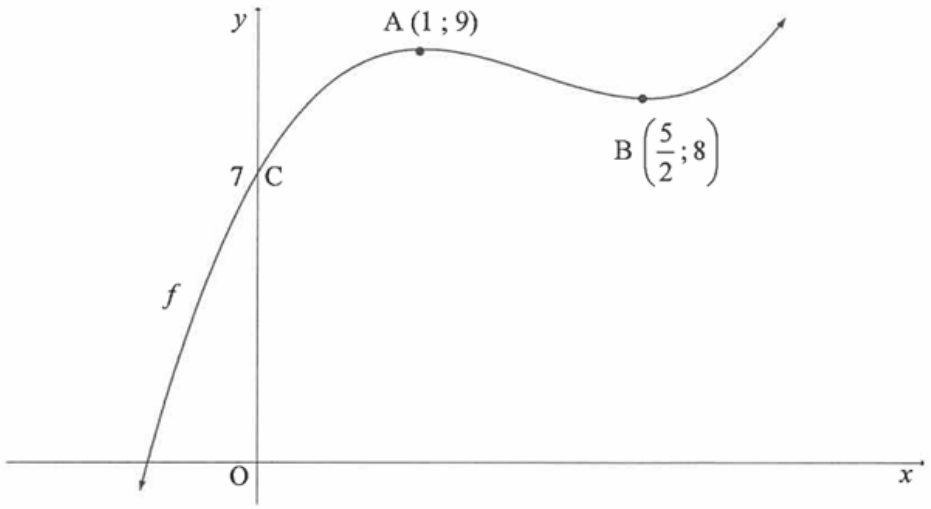
8.3.2 Write down how you will restrict the domain of f such that f^{-1}, the inverse of f, is a function. (1)

8.3.3 Determine the equation of f^{-1} for $f^{-1}(x) \leq 0$. Write your answer in the form $y = \ldots$ (3)
[18]

QUESTION 9

A(1 ; 9) and $B\left(\frac{5}{2};8\right)$ are the turning points of graph f below.

C(0 ; 7) is the y-intercept of f.



9.1 For which values of x is f decreasing? (2)

9.2 Write down the x-intercepts of f', the derivative of f. (2)

9.3 For which values of x will f be concave up? (2)

9.4 Determine the value of k for which $y = f(x) + k$ will have THREE positive x-intercepts. (2)
[8]

QUESTION 10

A cyclist rode from town P and stopped at town T. The speed (in km/h) at which this cyclist rode, is represented by the equation $s'(t) = -3t^2 + 18t$.

NOTE: Speed is the rate of change in distance with respect to time.

10.1 Calculate the maximum speed that the cyclist reached on this ride. (3)

10.2 Calculate the distance between town P and town T. (5)
[8]

Paper F

QUESTION 8

8.1 If $f(x) = 2x^2 - 5x + 3$, determine $f'(x)$ from first principles. (5)

8.2 Determine $\frac{dy}{dx}$ if $y = \frac{2x^2}{3\sqrt{x}} - \frac{2x^3 + 1}{x^3}$ (5)

[10]

QUESTION 9

9.1 Given: $f(x) = -2x^3 + 5x^2 + 4x - 3$

9.1.1 Calculate the coordinates of the x-intercepts of f if $f(3) = 0$. Show all calculations. (4)

9.1.2 Calculate the x-values of the stationary points of f. (4)

9.1.3 For which values of x is f concave up? (2)

9.2 The function g, defined by $g(x) = ax^3 + bx^2 + cx + d$ has the following properties:

- $g(-2) = g(4) = 0$
- The graph of $g'(x)$ is concave up.
- The graph of $g'(x)$ has x-intercepts at $x = 0$ and $x = 4$ and a turning point at $x = 2$.

9.2.1 Use this information to draw a neat sketch graph of g without actually solving for a, b, c and d. Clearly show all x-intercepts, x-values of the turning points and x-value of the point of inflection on your sketch. (4)

9.2.2 For which values of x will $g(x) \cdot g''(x) > 0$? (3)

[17]

QUESTION 10

A shopkeeper finds that the number of people visiting his shop at any moment during the 10 hours that the shop is open, is represented by:

$$N(t) = t^3 - 12t^2 + 36t + 8 \ ,$$

where $N(t)$ is the number of people in the shop, t hours after the shop opened.

10.1 How many people are in the shop when the shop opens? (1)

10.2 At what stage is the number of people in the shop increasing? (5)

10.3 At which stage is it the best time for the shopkeeper to take a break and leave his assistant alone in the shop? (1)
[7]

Paper G

QUESTION 7

7.1 Determine $f'(x)$ from first principles if $f(x) = x^2 - 5$. (5)

7.2 Determine the derivative of: $g(x) = 5x^2 - \frac{2x}{x^3}$ (3)

7.3 Given: $h(x) = ax^2$, $x > 0$.
Determine the value of a if it is given that $h^{-1}(8) = h'(4)$. (6)
[14]

QUESTION 8

Given: $f(x) = 2x^3 - 5x^2 + 4x$

8.1 Calculate the coordinates of the turning points of the graph of f. (5)

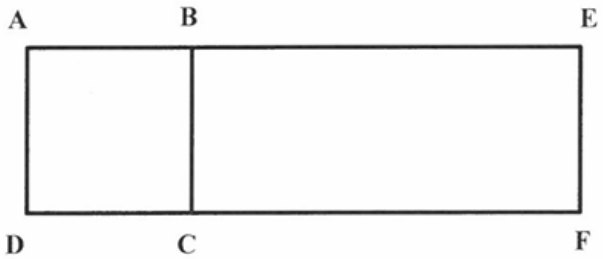
8.2 Prove that the equation $2x^3 - 5x^2 + 4x = 0$ has only one real root. (3)

8.3 Sketch the graph of f, clearly indicating the intercepts with the axes and the turning points. (3)

8.4 For which values of x will the graph of f be concave up? (3)
[14]

QUESTION 9

A piece of wire 6 metres long is cut into two pieces. One piece, x metres long, is bent to form a square ABCD. The other piece is bent into a U-shape so that it forms a rectangle BEFC when placed next to the square, as shown in the diagram below.



Calculate the value of x for which the sum of the areas enclosed by the wire will be a maximum. **[7]**

Paper H

QUESTION 8

8.1 If $f(x) = x^2 - 3x$, determine $f'(x)$ from first principles. (5)

8.2 Determine:

8.2.1 $\frac{dy}{dx}$ if $y = \left(x^2 - \frac{1}{x^2}\right)^2$ (3)

8.2.2 $D_x\left(\frac{x^3 - 1}{x - 1}\right)$ (3)
[11]

QUESTION 9

Given: $h(x) = -x^3 + ax^2 + bx$ and $g(x) = -12x$. P and Q(2 ; 10) are the turning points of h. The graph of h passes through the origin.

9.1 Show that $a = \frac{3}{2}$ and $b = 6$. (5)

9.2 Calculate the average gradient of h between P and Q, if it is given that $x = -1$ at P. (4)

9.3 Show that the concavity of h changes at $x = \frac{1}{2}$. (3)

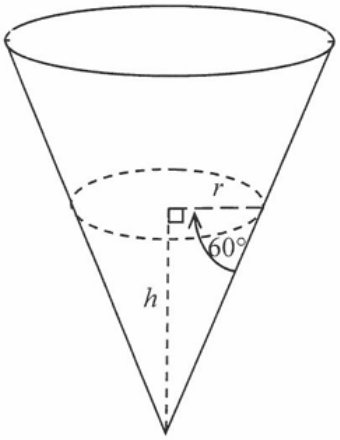
9.4 Explain the significance of the change in QUESTION 9.3 with respect to h. (1)

9.5 Determine the value of x, given $x < 0$, at which the tangent to h is parallel to g. (4)

[17]

QUESTION 10

A rain gauge is in the shape of a cone. Water flows into the gauge. The height of the water is h cm when the radius is r cm. The angle between the cone edge and the radius is 60°, as shown in the diagram below.



Formulae for volume:

$V = \pi r^2 h$ $\quad V = \frac{1}{3}\pi r^2 h$

$V = lbh$ $\quad V = \frac{4}{3}\pi r^3$

10.1 Determine r in terms of h. Leave your answer in surd form. (2)

10.2 Determine the derivative of the volume of water with respect to h when h is equal to 9 cm. (5)

[7]

Paper I

QUESTION 7

7.1 Determine $f'(x)$ from first principles if $f(x) = x^2 + x$. (5)

7.2 Determine $f'(x)$ if $f(x) = 2x^5 - 3x^4 + 8x$. (3)

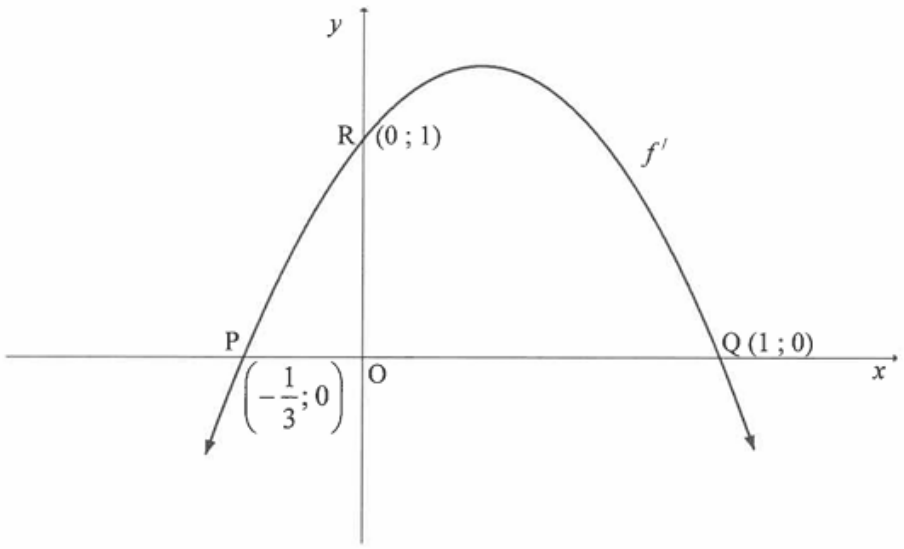
7.3 The tangent to $g(x) = ax^3 + 3x^2 + bx + c$ has a minimum gradient at the point $(-1\,;-7)$. For which values of x will g be concave up? (4)

[12]

QUESTION 8

The graph of $y = f'(x) = mx^2 + nx + k$ is drawn below.

The graph passes the points $P\left(-\frac{1}{3}; 0\right)$, $Q(1; 0)$ and $R(0; 1)$.



8.1 Determine the values of m, n and k. (6)

8.2 If it is further given that $f(x) = -x^3 + x^2 + x + 2$:

8.2.1 Determine the coordinates of the turning points of f. (3)

8.2.2 Draw the graph of f. Indicate on your graph the coordinates of the turning points and the intercepts with the axes. (5)

8.3 Points E and W are two variable points on f' and are on the same horizontal line.

- h is a tangent to f' at E.
- g is a tangent to f' at W.
- h and g intersect at D(a ; b).

8.3.1 Write down the value of a. (1)

8.3.2 Determine the value(s) of b for which h and g will no longer be tangents to f'. (2)

[17]

QUESTION 9

Given $f(x) = x^2$.

Determine the minimum distance between the point $(10 ; 2)$ and a point on f. **[8]**

Paper J

QUESTION 7

7.1 Determine $f'(x)$ from first principles if it is given that $f(x) = 4 - 7x$. (4)

7.2 Determine $\frac{dy}{dx}$ if $y = 4x^8 + \sqrt{x^3}$ (3)

7.3 Given: $y = ax^2 + a$

Determine:

7.3.1 $\frac{dy}{dx}$ (1)

7.3.2 $\frac{dy}{da}$ (2)

7.4 The curve with equation $y = x + \frac{12}{x}$ passes through the point A(2 ; b). Determine the equation of the line perpendicular to the tangent to the curve at A. (4)
[14]

QUESTION 8

After flying a short distance, an insect came to rest on a wall. Thereafter the insect started crawling on the wall. The path that the insect crawled can be described by $h(t) = (t-6)(-2t^2 + 3t - 6)$, where h is the height (in cm) above the floor and t is the time (in minutes) since the insect started crawling.

8.1 At what height above the floor did the insect start to crawl? (1)

8.2 How many times did the insect reach the floor? (3)

8.3 Determine the maximum height that the insect reached above the floor. (4)
[8]

QUESTION 9

Given: $f(x) = 3x^3$

9.1 Solve $f(x) = f'(x)$ (3)

9.2 The graphs f, f' and f'' all pass through the point $(0\ ;\ 0)$.

9.2.1 For which of the graphs will $(0\ ;\ 0)$ be a stationary point? (1)

9.2.2 Explain the difference, if any, in the stationary points referred to in QUESTION 9.2.1. (2)

9.3 Determine the vertical distance between the graphs of f' and f'' at $x = 1$. (3)

9.4 For which value(s) of x is $f(x) - f'(x) < 0$? (4)

[13]

PROBABILITY AND COUNTING PRINCIPLES

Paper A

QUESTION 11

11.1 A survey was conducted among female and male learners at a school about which type of cold drink they preferred. The data from the survey is presented in the table below.

	JUICE	ENERGY DRINKS	TOTAL
Female	*a*	*b*	*c*
Male	36	54	*f*
Total	*e*	*d*	210

11.1.1 The events male and preferring juice are independent. Show that $e = 84$. (3)

11.1.2 Calculate the probability that a female learner, chosen at random from the group, will like energy drinks. (3)

11.2 At a kiosk, 120 people buy either a cup of coffee or a bottle of water. The chance of rain on any given day is 75%. The chance of a person buying a cup of coffee on a rainy day is three times the chance of the person buying coffee on a non-rainy day.

The probability of a person buying coffee on any given day is $\frac{7}{12}$.

Calculate the number of cups of coffee that will be sold on a non-rainy day. (4)

11.3 Eight runners compete in a race where there are no tied finishes. Bongi and Andrew are two of the competitors.

11.3.1 Calculate the total number of possible ways in which the 8 runners can finish the race if Bongi finishes in a position immediately after Andrew. (2)

11.3.2 Calculate the probability that TWO OR MORE runners finish the race after Andrew and before Bongi. (4)

[16]

Paper B

QUESTION 10

10.1 A and B are mutually exclusive events.
If P(A) = 0,42 and P(A or B) = 0,79, calculate P(B). (2)

10.2 A game at a fun park requires a player to roll a six-sided dice and pick a card from a deck of 52 cards.

A player wins if an odd number appears on the uppermost face of the dice and the player also draws a picture card from the deck.

- A deck of cards has 4 suites (hearts, diamonds, spades and clubs).
- There are 4 picture cards (king, queen, jack and ace) in every suite.

A player pays R10 to play a game and in an hour, 260 people can each play one game. If the owner wants to make a 70% profit per hour, calculate the maximum amount that the owner must pay out to each winner. (6)
[8]

QUESTION 11

Consider the three-digit numbers from 501 up to 999.

11.1 How many of these three-digit numbers have exactly one 5 in them? (4)

11.2 Calculate the probability of a three-digit number not satisfying the condition given in QUESTION 11.1. (3)
[7]

Paper C

QUESTION 11

A certain number of learners are sitting for examinations in Mathematics, Tourism and Geography.

- All these learners sit for at least one of these examinations.
- The total number of learners who sit for Mathematics (M), is 22.
- The total number of learners sitting for Tourism (T), is 16.
- The total number of learners sitting for Geography (G), is 18.
- 5 learners sit for Mathematics and Tourism, but not Geography.
- 4 learners sit for Mathematics and Geography, but not Tourism.
- 3 learners sit for Tourism and Geography, but not Mathematics.
- 6 learners sit for only Tourism.

11.1 Draw a Venn diagram to represent ALL the learners sitting for these examinations. (3)

11.2 Calculate the probability that a learner, chosen at random, will sit for examinations in at least TWO of the subjects. (2)

11.3 Determine if the events: sitting for examinations in Mathematics and sitting for examinations in Tourism are independent. Support your answer with the necessary calculations. (4)
[9]

QUESTION 12

A company generates a 4-character code using the 26 letters of the alphabet and the 10 digits, from 0 to 9.

The code is in the form:

letter	digit	letter	digit

12.1 Determine how many different codes can be formed if letters and digits may be repeated. (2)

12.2 Determine how many different codes can be formed if:

- The letters D, F, I, Q, U and V may NOT be used
- The code may NOT start with a W or a Z
- Letters or digits may NOT be repeated
- The code ends with an odd digit (4)

12.3 The company wishes to increase the number of 4-character codes formed in QUESTION 12.2 by allowing the letters D, F, I, Q, U and V to be used. Calculate the percentage increase in the number of different codes that can now be formed. (2)

[8]

Paper D

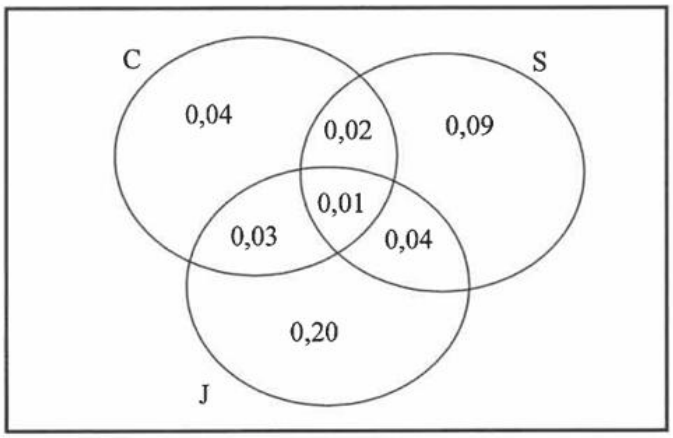
QUESTION 11

11.1 Two events, A and B, are such that:

- P(A) = 0,4
- P(A or B) = 0,52
- A and B are mutually exclusive

Calculate P(B). (2)

11.2 The items that a learner bought at a tuck shop were recorded over a period of time. The probabilities of the learner buying a sandwich (S), a chocolate (C) and a juice (J) are shown in the Venn diagram below.



11.2.1 What is the probability that the learner will buy a sandwich? (1)

11.2.2 Calculate the probability that the learner will buy at least two of the three items. (2)

11.2.3 Calculate the probability that the learner would NOT buy any of the three items. (2)

11.3 Seven guitar players, each with a different name, participate in a concert.

11.3.1 In how many different ways can the names of the guitar players be listed, one below the other, in the programme? (1)

11.3.2 After the performance, the guitar players wait backstage. There is a bench with only room for four to sit on.

What will be the probability that the four guitar players will be sitting in alphabetical order, from left to right? (3)

11.3.3 During the performance, the seven guitar players sit in a line on stage. Four guitar players are female and three are male.

In how many different ways can they be seated if the males may not sit next to each other? (3)

[14]

Paper E

QUESTION 10

10.1 A and B are independent events. $P(A) = \frac{1}{3}$ and $P(B) = \frac{3}{4}$

Determine:

10.1.1 P(A and B) (2)

10.1.2 P(at least ONE event occurs) (2)

10.2 The probability that it will snow on the Drakensberg Mountains in June is 5%.

- When it snows on the mountains, the probability that the minimum temperature in Central South Africa will drop below 0 °C is 72%.
- If it does not snow on the mountains, the probability that the minimum temperature in Central South Africa will drop below 0 °C is 35%.

10.2.1 Represent the given information on a tree diagram. Clearly indicate the probabilities associated with EACH branch. (3)

10.2.2 Calculate the probability that the temperature in Central South Africa will NOT drop below 0 °C in June 2024. (3)

10.3 Ten learners stand randomly in a line, one behind the other.

10.3.1 In how many different ways can the ten learners stand in the line? (1)

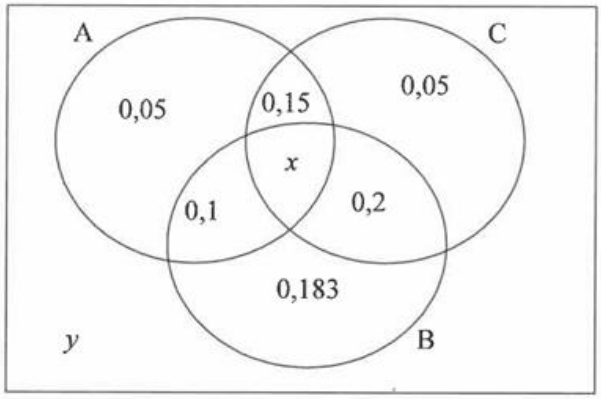
10.3.2 Calculate the probability that there will be 5 learners between the 2 youngest learners in the line. (4)

[15]

Paper F

QUESTION 10

10.1 A, B and C are three events. The probabilities of these events (or any combination of them) occurring is given in the Venn-diagram below



10.1.1 If it is given that the probability that at least one of the events will occur is 0,893, calculate the value of:

(a) y, the probability that none of the events will occur. (1)

(b) x, the probability that all three events will occur. (1)

10.1.2 Determine the probability that at least two of the events will take place. (2)

10.1.3 Are events B and C independent? Justify your answer. (5)

10.2 A four-digit code is required to open a combination lock. The code must be even-numbered and may not contain the digits 0 or 1. Digits may not be repeated.

10.2.1 How many possible 4-digit combinations are there to open the lock? (3)

10.2.2 Calculate the probability that you will open the lock at the first attempt if it is given that the code is greater than 5 000 and the third digit is 2. (5)
[17]

Paper G

QUESTION 10

The school library is open from Monday to Thursday. Anna and Ben both studied in the school library one day this week. If the chance of studying any day in the week is equally likely, calculate the probability that Anna and Ben studied on:

10.1 The same day (2)

10.2 Consecutive days (3)
[5]

QUESTION 11

11.1 Events **A** and **B** are independent. P(A) = 0,4 and P(B) = 0,25.

11.1.1 Represent the given information on a Venn diagram. Indicate on the Venn diagram the probabilities associated with each region. (3)

11.1.2 Determine P(**A** or NOT **B**). (2)

11.2 Motors Incorporated manufacture cars with 5 different body styles, 4 different interior colours and 6 different exterior colours, as indicated in the table below.

BODY STYLES	INTERIOR COLOURS	EXTERIOR COLOURS
Five body styles	Blue	Silver
		Blue
	Grey	White
	Black	Green
		Red
	Red	Gold

The interior colour of the car must NOT be the same as the exterior colour.

Motors Incorporated wants to display one of each possible variation of its car in their showroom. The showroom has a floor space of 500 m^2 and each car requires a floor space of 5 m^2.

Is this display possible? Justify your answer with the necessary calculations. (6)
[11]

Paper H

QUESTION 10

In a certain country, 10-digit telephone numbers with the following format were introduced:

Format	Area Code	Exchange Code	Number
Number of digits	3 digits	3 digits	4 digits
Example	901	544	1230

Digits may be repeated.

10.1 How many possible 10-digit telephone numbers could be formed? (2)

10.2 Certain restrictions were placed on the groups of digits:

- Area code: must be 3 digits and the first digit can NOT be 0 or 1
- Exchange code: must be 3 digits and the first and second digits can NOT be 0 or 1
- Number: must be 4 digits and the first digit MUST be a 0 or 1

10.2.1 How many valid 10-digit telephone numbers could be formed by applying the given restrictions? (3)

10.2.2 Determine the probability that any randomly chosen 10-digit telephone number would be a valid phone number. (2)

[7]

QUESTION 11

Harry shoots arrows at a target board. He has a 50% chance of hitting the bull's eye on each shot.

11.1 Calculate the probability that Harry will hit the bull's eye in his first shot and his second shot. (2)

11.2 Calculate the probability that Harry will hit the bull's eye at least twice in his first three shots. (3)

11.3 Glenda also has a 50% chance of hitting the bull's eye on each shot. Harry and Glenda will take turns to shoot an arrow and the first person to hit the bull's eye will be the winner. Calculate the probability that the person who shoots first will be the winner of the challenge. (3)

[8]

Paper I

QUESTION 11

11.1 For two events, A and B, it is given that:

$P(A) = 0,2$
$P(B) = 0,63$
$P(A \text{ and } B) = 0,126$

Are the events, A and B, independent? Justify your answer with appropriate calculations. (3)

11.2 The letters of the word DECIMAL are randomly arranged into a new 'word', also consisting of seven letters. How many different arrangements are possible if:

11.2.1 Letters may be repeated (2)

11.2.2 Letters may not be repeated (2)

11.2.3 The arrangements must start with a vowel and end in a consonant and no repetition of letters is allowed (4)

11.3 There are t orange balls and 2 yellow balls in a bag. Craig randomly selects one ball from the bag, records his choice and returns the ball to the bag. He then randomly selects a second ball from the bag, records his choice and returns it to bag. It is known that the probability that Craig will select two balls of the same colour from the bag is 52%.

Calculate how many orange balls are in the bag. (6)
[17]

Paper J

QUESTION 10

10.1 The events S and T are independent.

- $P(S \text{ and } T) = \frac{1}{6}$
- $P(S) = \frac{1}{4}$

10.1.1 Calculate $P(T)$. (2)

10.1.2 Hence, calculate $P(S \text{ or } T)$. (2)

10.2 A FIVE-digit code is created from the digits 2 ; 3 ; 5 ; 7 ; 9.

How many different codes can be created if:

10.2.1 Repetition of digits is NOT allowed in the code (2)

10.2.2 Repetition of digits IS allowed in the code (1)

10.3 A group of 3 South Africans, 2 Australians and 2 Englishmen are staying at the same hotel while on holiday. Each person has his/her own room and the rooms are next to each other in a straight corridor.

If the rooms are allocated at random, determine the probability that the 2 Australians will have adjacent rooms and the 2 Englishmen will also have adjacent rooms. (4)

[11]

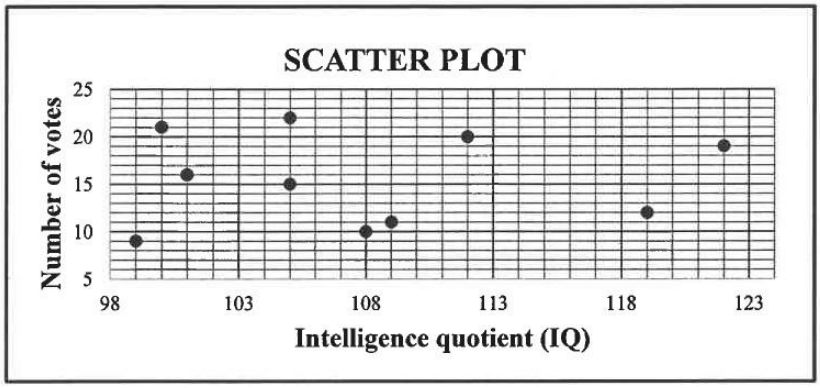
SECTION B: PAPER 2

STATISTICS

Paper A

QUESTION 1

The matric class of a certain high school had to vote for the chairperson of the RCL (representative council of learners). The scatter plot below shows the IQ (intelligence quotient) of the 10 learners who received the most votes and the number of votes that they received.



Before the election, the popularity of each of these ten learners was established and a popularity score (out of a 100) was assigned to each. The popularity scores and the number of votes of the same 10 learners who received the most votes are shown in the table below.

Popularity score (x)	32	89	35	82	50	59	81	40	79	65
Number of votes (y)	9	22	10	21	11	15	20	12	19	16

1.1 Calculate the:

1.1.1 Mean number of votes that these 10 learners received (2)

1.1.2 Standard deviation of the number of votes that these 10 learners received (1)

1.2 The learners who received fewer votes than one standard deviation below the mean were not invited for an interview. How many learners were invited? (2)

1.3 Determine the equation of the least squares regression line for the data given in the table. (3)

1.4 Predict the number of votes that a learner with a popularity score of 72 will receive. (2)

1.5 Using the scatter plot and table above, provide a reason why:

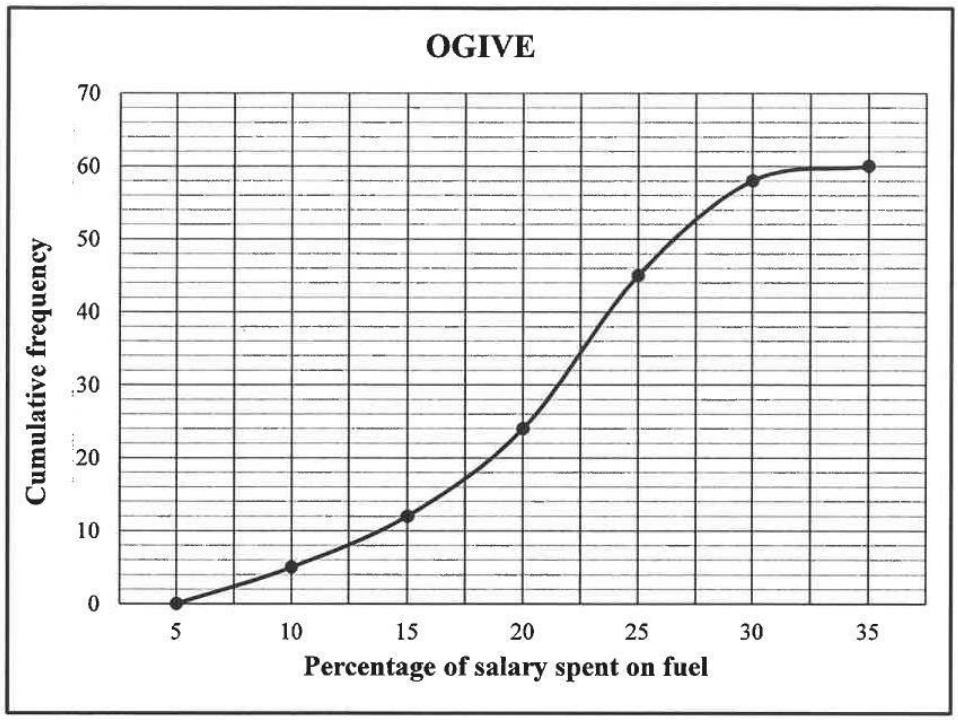
1.5.1 IQ is not a good indicator of the number of votes that a learner could receive (1)

1.5.2 The prediction in QUESTION 1.4 is reliable (1)

[12]

QUESTION 2

A company conducted research among all its employees on what percentage of their monthly salary was spent on fuel in a particular month. The data is represented in the ogive (cumulative frequency graph) below.



2.1 How many people are employed at this company? (1)

2.2 Write down the modal class of the data. (1)

2.3 How many employees spent more than 22,5% of their monthly salary on fuel? (2)

2.4 An employee spent R2 400 of his salary on fuel in that particular month. Determine the monthly salary of this employee if he spends 7% of his salary on fuel. (2)

2.5 The monthly salaries of these employees remains constant and the number of litres of fuel used in each month also remains constant. If the fuel price increases from R21,43 per litre to R22,79 per litre at the beginning of the next month, how will the above ogive change? (2)
[8]

Paper B

QUESTION 1

Truck drivers travel a certain distance and have a rest before travelling further. A driver kept record of the distance he travelled (in km) on 8 trips and the amount of time he rested (in minutes) before he continued his journey. The information is given in the table below.

Distance travelled (in km) (x)	180	200	400	600	170	350	270	300
Amount of rest time (in minutes) (y)	20	25	55	120	15	50	40	45

1.1 Determine the equation of the least squares regression line for the data. (3)

1.2 If a truck driver travelled 550 km, predict the amount of time (in minutes) that he should rest before continuing his journey. (2)

1.3 Write down the correlation coefficient for the data. (1)

1.4 Interpret your answer to QUESTION 1.3. (1)

1.5 At each stop, the truck driver spent money buying food and other refreshments. The amount spent (in rands) is given in the table below.

100	150	130	200	50	180	200	190

1.5.1 Calculate the mean amount of money he spent at each stop. (2)

1.5.2 Calculate the standard deviation for the data. (1)

1.5.3 At how many stops did the driver spend an amount that was less than one standard deviation below the mean? (2)

[12]

QUESTION 2

At a certain school, the staff committee wanted to determine how many glasses of water the staff members drank during a school day. All teachers present on a specific day were interviewed. The information is shown in the table below.

NUMBER OF GLASSES OF WATER DRANK PER DAY	NUMBER OF STAFF MEMBERS
$0 \le x < 2$	5
$2 \le x < 4$	15
$4 \le x < 6$	13
$6 \le x < 8$	5
$8 \le x < 10$	2

2.1 Complete the cumulative frequency column provided in the table in the ANSWER BOOK. (2)

2.2 How many staff members were interviewed? (1)

2.3 How many staff members drank fewer than 6 glasses of water during a school day? (1)

2.4 The staff committee observed that k teachers were absent on the day of the interviews. It was found that half of these k teachers drank from 0 to fewer than 2 (that is $0 \le x < 2$) glasses of water per day, while the remainder of them drank from 4 to fewer than 6 (that is $4 \le x < 6$) glasses of water per day. When these k teachers are included in the data, the estimated mean is 4 glasses of water per staff member per day.

How many teachers were absent on the day of the interviews? (4)
[8]

Paper C

QUESTION 1

The table below shows the mass (in kg) of the school bags of 80 learners.

MASS (in kg)	FREQUENCY
$5 < m \leq 7$	6
$7 < m \leq 9$	18
$9 < m \leq 11$	21
$11 < m \leq 13$	19
$13 < m \leq 15$	11
$15 < m \leq 17$	4
$17 < m \leq 19$	1

1.1 Write down the modal class of the data. (1)

1.2 Complete the cumulative frequency column in the table in the ANSWER BOOK. (2)

1.3 Draw a cumulative frequency graph (ogive) for the given data on the grid provided in the ANSWER BOOK. (3)

1.4 Use the graph to determine the median mass for this data. (2)

1.5 The international guideline for the mass of a school bag is that it should not exceed 10% of a learner's body mass.

1.5.1 Calculate the estimated mean mass of the school bags. (2)

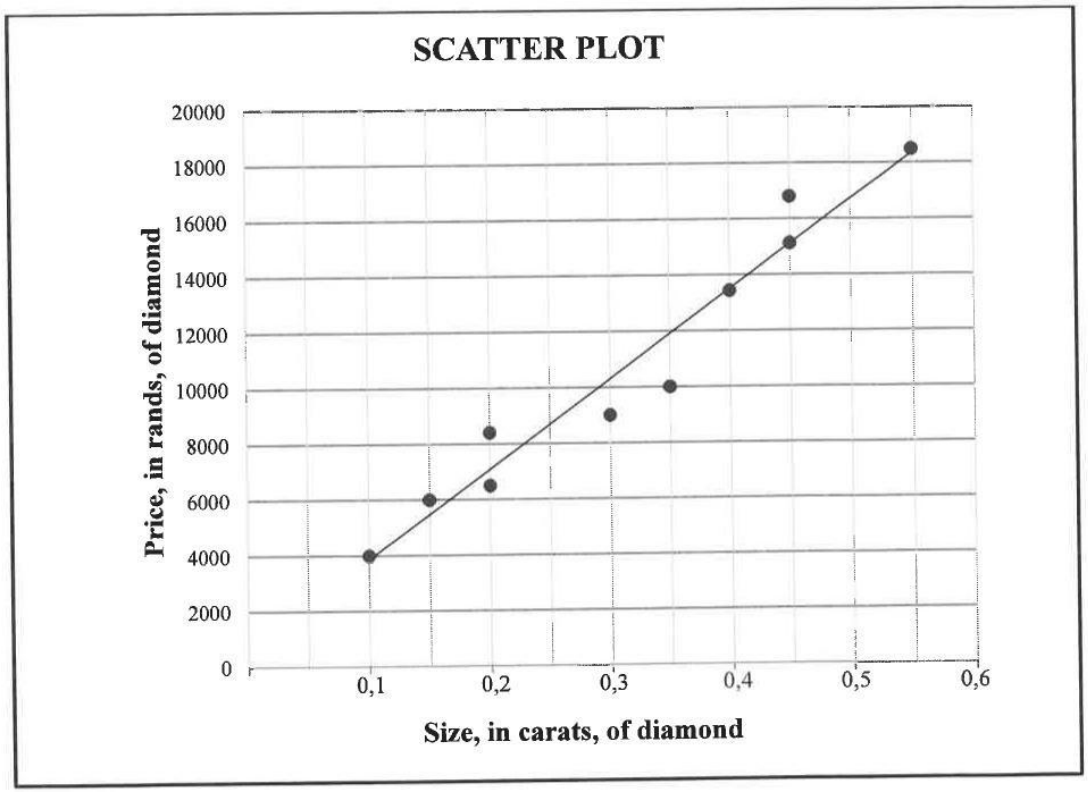
1.5.2 The mean mass of this group of learners was found to be 80 kg. On average, are these school bags satisfying the international guideline with regard to mass? Motivate your answer. (2)

[12]

QUESTION 2

The table below shows the size (in carats) and the price (in rands) of 10 diamonds that were sold by a diamond trader. This information is also presented in the scatter plot below. The least squares regression line for the data is drawn.

Size, in carats, of diamond (x)	0,1	0,15	0,2	0,2	0,3	0,35	0,4	0,45	0,45	0,55
Price, in rands, of diamond (y)	4 000	6 000	6 500	8 400	9 000	10 000	13 440	15 120	16 800	18 480



2.1 Determine the equation of the least squares regression line for the data. (3)

2.2 If the trader sold a diamond that was 0,25 carats in size, predict the selling price of this diamond in rands. (2)

2.3 Calculate the average price increase per 0,05 carat of the diamonds. (2)

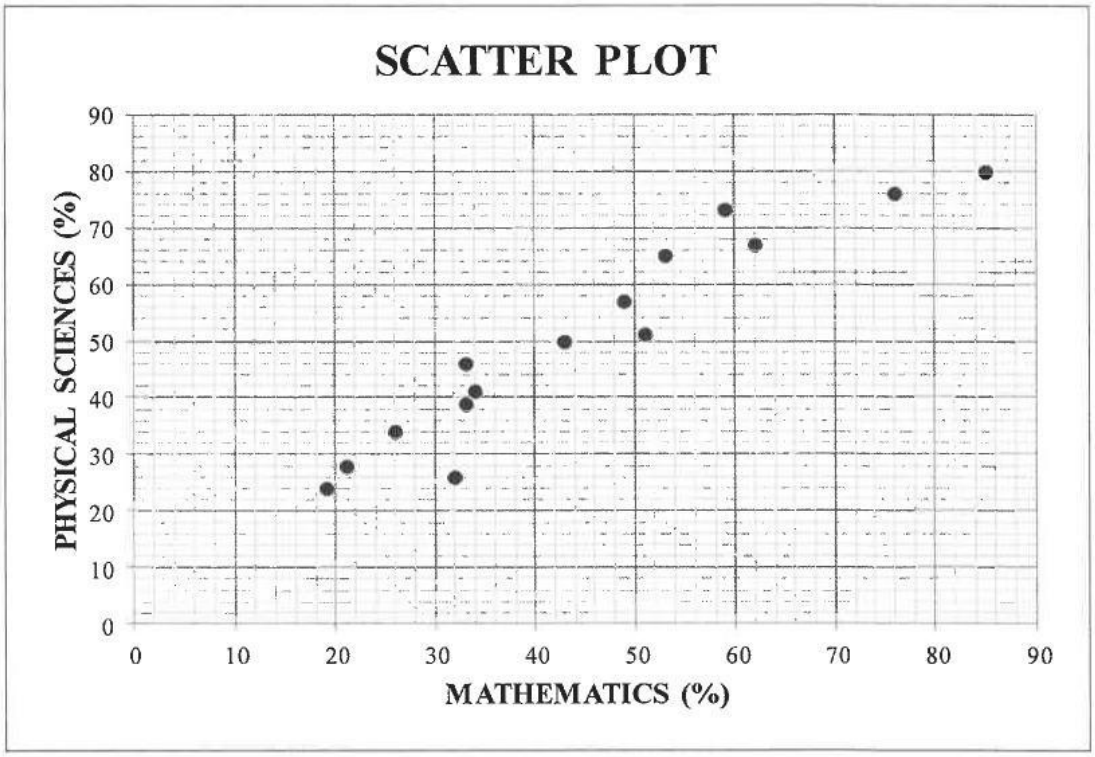
2.4 It was later found that the selling price of the 0,35 carat diamond was recorded incorrectly. The correct price is R11 500. When this correction is made to the data set, the correlation between the size and price of these diamonds gets stronger. Explain the reason for this by referring to the given scatter plot. (1)
[8]

Paper D

QUESTION 1

A Mathematics teacher was curious to establish if her learners' Mathematics marks influenced their Physical Sciences marks. In the table below, the Mathematics and Physical Sciences marks of 15 learners in her class are given as percentages (%).

MATHEMATICS (AS %)	26	62	21	33	53	76	32	59	43	33	49	51	19	34	85
PHYSICAL SCIENCES (AS %)	34	67	28	46	65	76	26	73	50	39	57	51	24	41	80



1.1 Determine the equation of the least squares regression line for the data. (3)

1.2 Draw the least squares regression line on the scatter plot provided in the ANSWER BOOK. (2)

1.3 Predict the Physical Sciences mark of a learner who achieved 69% for Mathematics. (2)

1.4 Write down the correlation coefficient between the Mathematics and Physical Sciences marks for the data. (1)

1.5 Comment on the strength of the correlation between the Mathematics and Physical Sciences marks for the data. (1)

1.6 What trend did the teacher observe between the results of the two subjects? (1)

[10]

QUESTION 2

The number of aircraft landing at the King Shaka International Airport and the Port Elizabeth Airport for the period starting in April 2017 and ending in March 2018, is shown in the double bar graph below.



[Source: http://www.airports.co.za/airports/statistics/aircraft]

2.1 The number of aircraft landing at the Port Elizabeth Airport exceeds the number of aircraft landing at the King Shaka International Airport during some months of the given period. During which month is this difference the greatest? (1)

2.2 The number of aircraft landing at the King Shaka International Airport during these months are:

2 182	2 323	2 267	2 334	2 346	2 175
2 293	2 263	2 215	2 271	2 018	2 254

Calculate the mean for the data. (2)

2.3 Calculate the standard deviation for the number of aircraft landing at the King Shaka International Airport for the given period. (2)

2.4 Determine the number of months in which the number of aircraft landing at the King Shaka International Airport were within one standard deviation of the mean. (3)

2.5 Which ONE of the following statements is CORRECT?

A. During December and January, there were more landings at the Port Elizabeth Airport than at the King Shaka International Airport.

B. There was a greater variation in the number of aircraft landing at the King Shaka International Airport than at the Port Elizabeth Airport for the given period.

C. The standard deviation of the number of landings at the Port Elizabeth Airport will be higher than the standard deviation of the number of landings at the King Shaka International Airport. (1)

[9]

Paper E

QUESTION 1

There were 11 cars of the same model for sale at a car dealership. The age (in years) of each car and its corresponding selling price (in rands) is provided in the table below.

AGE OF CAR (IN YEARS)	SELLING PRICE OF CAR (IN RAND)
2	293 000
3	265 000
3	256 000
4	219 000
4	241 000
4	246 000
6	226 000
6	176 000
7	154 000
7	180 000
8	148 000

1.1 Determine the equation of the least squares regression line. (3)

1.2 Predict the selling price of a similar car at this car dealership that is 5 years old. (2)

1.3 Use the correlation coefficient to show whether the prediction made in QUESTION 1.2 is valid or not. (2)

1.4 Use the answer to QUESTION 1.1 to write down the estimated average yearly decrease in the selling price of these 11 cars. (1)
[8]

QUESTION 2

2.1 The cumulative frequency table below shows the amount of time that people spent on a particular website on a certain day.

TIME, t (IN MINUTES)	CUMULATIVE FREQUENCY
$0 < t \leq 20$	16
$0 < t \leq 40$	40
$0 < t \leq 60$	59
$0 < t \leq 80$	67
$0 < t \leq 100$	70

2.1.1 How many people visited this website on that day? (1)

2.1.2 How many people spent more than 40 and up to 80 minutes on the website? (2)

2.1.3 Draw a histogram to represent the information provided in the cumulative frequency table. (3)

2.1.4 Comment on the skewness of the data. (1)

2.2 There are 9 players in a basketball team. The coach calculated that on average, each player scored 12 points during a game. The points scored by 8 of the 9 players from the team is given below:

11	14	19	20	8	10	2	14

How many players' points score was outside ONE standard deviation of the mean points score? (5)

[12]

Paper F

QUESTION 1

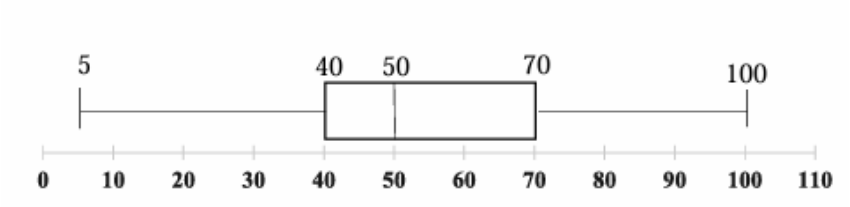
The A-Rithmetic High School decided to compare the results of 31 Grade 12 learners in Mathematics and Physical Sciences in the 2019 Preparatory Examination.

- The Mathematics results are recorded in the table below.
- The box and whisker plot below illustrates the results of Physical Sciences.
- Marks are recorded as percentages.

Mathematics Results

7	11	15	19	19	23	28	28	31	38	39
40	41	48	48	52	53	55	57	59	59	64
67	72	76	83	85	87	89	92	96	-	-

Physical Sciences Results



1.1 Calculate the mean mark of the Mathematics learners. (2)

1.2 Comment on the skewness of the Mathematics data. (1)

1.3 Determine which subject performed better in the 2019 Preparatory Examination. Give a reason for your conclusion. (2)

1.4 Write down a possible mark for a learner who achieved the tenth lowest mark in Physical Sciences. (2)

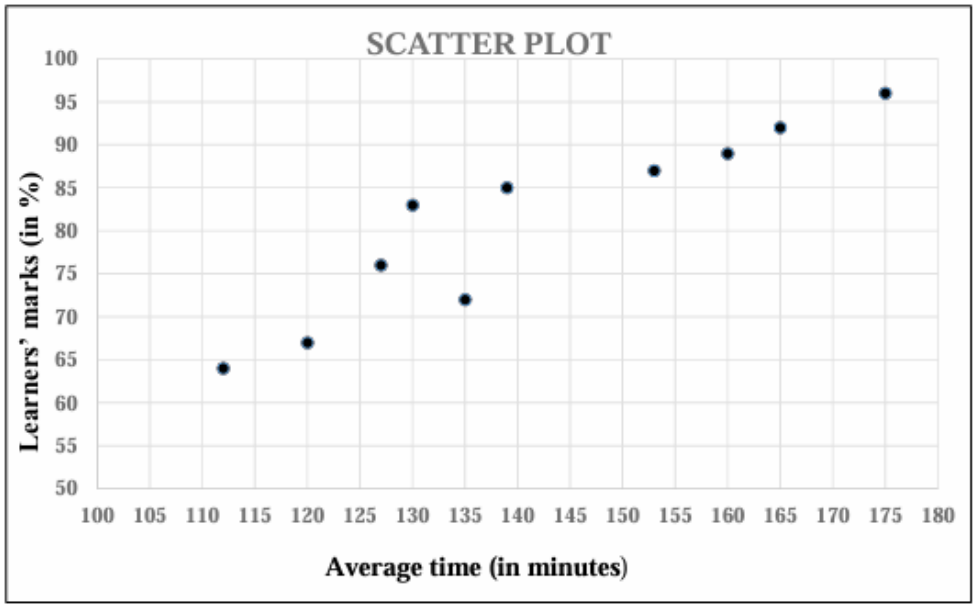
1.5 A learner scored the fourth highest in both subjects. The learner obtained the GREATEST possible difference between both subjects. Calculate the learner's mark in Physical Sciences. (2)
[9]

QUESTION 2

A question raised by many educators is whether the results that a learner achieves in an examination is dependent on the time that the learner takes to complete the examination.

The average time taken by each of the top 10 Mathematics learners was recorded. The data is represented in the table and scatter plot below.

Average time (in minutes)	175	165	160	153	139	130	127	135	120	112
Learners' marks (in %)	96	92	89	87	85	83	76	72	67	64



2.1 Calculate the equation of the least squares regression line for the data. (3)

2.2 A learner completed the exam in 2,5 hours. Predict the mark that the learner achieved. (2)

2.3 Explain within the context why the regression line is not reliable. (1)

2.4 Calculate the standard deviation of the top 10 Mathematics learners. (2)

2.5 It is further given that $(p\,;103,59)$ is the interval of 15 random learners' marks within ONE standard deviation of the mean. If $\bar{x} = 63{,}96$, calculate the value of p. (3)

[11]

Paper G

QUESTION 1

A bakery kept a record of the number of loaves of bread a tuck-shop ordered daily over the last 18 days. The information is shown in the table below.

10	11	13	14	14	15	16	18	18
19	19	20	21	35	35	37	40	41

1.1 Calculate the:

1.1.1 Mean number of loaves of bread ordered daily (2)

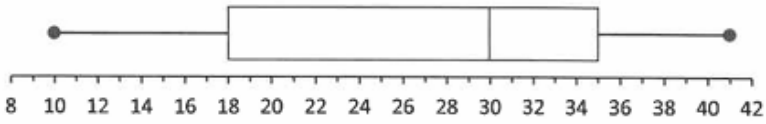
1.1.2 Standard deviation of the data (1)

1.1.3 Number of days on which the number of loaves of bread ordered was more than one standard deviation above the mean (2)

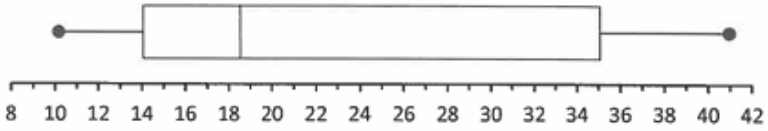
1.2 The tuck-shop owner was not able to sell all the loaves of bread delivered daily. He calculated the mean number of loaves sold over the 18 days to be 20. Calculate the number of loaves of bread which were NOT sold over the 18 days. (2)

1.3 One of the two box and whisker diagrams drawn below represents the data given in the table above.

Graph A:



Graph B:



1.3.1 Which ONE of the two box and whisker diagrams, drawn above, correctly represents the data? Write down a reason for your answer. (2)

1.3.2 Describe the skewness of the data. (1)

[10]

QUESTION 2

A farm stall sells milk in 5-litre containers to the local community. The price varies according to the availability of milk at the farm stall. The price of milk, in rands per 5-litre container, and the number of 5-litre containers of milk sold, are recorded in the table below.

Price of milk in rands per 5-litre container (x)	26	32	36	28	40	33	29	34	27	30
Number of 5-litre containers of milk sold (y)	48	30	26	44	23	32	39	29	42	33

2.1 On the grid provided in the ANSWER BOOK, draw the scatter plot to represent the data. (3)

2.2 Determine the equation of the least squares regression line for the data. (3)

2.3 If the farmer sells a 5-litre container of milk for R38, predict the number of 5-litre containers of milk he will sell. (2)

2.4 Refer to the correlation between the price of 5-litre containers of milk and the number of 5-litre containers of milk sold, and comment on the accuracy of your answer to QUESTION 2.3. (2)

[10]

Paper H

QUESTION 1

Each child in a group of four-year-old children was given the same puzzle to complete. The time taken (in minutes) by each child to complete the puzzle is shown in the table below.

TIME TAKEN (t) (IN MINUTES)	NUMBER OF CHILDREN
$2 < t \leq 6$	2
$6 < t \leq 10$	10
$10 < t \leq 14$	9
$14 < t \leq 18$	7
$18 < t \leq 22$	8
$22 < t \leq 26$	7
$26 < t \leq 30$	2

1.1 How many children completed the puzzle? (1)

1.2 Calculate the estimated mean time taken to complete the puzzle. (2)

1.3 Complete the cumulative frequency column in the table given in the ANSWER BOOK. (2)

1.4 Draw a cumulative frequency graph (ogive) to represent the data on the grid provided in the ANSWER BOOK. (3)

1.5 Use the graph to determine the median time taken to complete the puzzle. (2)

[10]

QUESTION 2

Learners who scored a mark below 50% in a Mathematics test were selected to use a computer-based programme as part of an intervention strategy. On completing the programme, these learners wrote a second test to determine the effectiveness of the intervention strategy. The mark (as a percentage) scored by 15 of these learners in both tests is given in the table below.

LEARNER	L1	L2	L3	L4	L5	L6	L7	L8	L9	L10	L11	L12	L13	L14	L15
TEST 1 (%)	10	18	23	24	27	34	34	36	37	39	40	44	45	48	49
TEST 2 (%)	33	21	32	20	58	43	49	48	41	55	50	45	62	68	60

2.1 Determine the equation of the least squares regression line. (3)

2.2 A learner's mark in the first test was 15 out of a maximum of 50 marks.

2.2.1 Write down the learner's mark for this test as a percentage. (1)

2.2.2 Predict the learner's mark for the second test. Give your answer to the nearest integer. (2)

2.3 For the 15 learners above, the mean mark of the second test is 45,67% and the standard deviation is 13,88%. The teacher discovered that he forgot to add the marks of the last question to the total mark of each of these learners. All the learners scored full marks in the last question. When the marks of the last question are added, the new mean mark is 50,67%.

2.3.1 What is the standard deviation after the marks for the last question are added to each learner's total? (2)

2.3.2 What is the total mark of the last question? (2)

[10]

Paper I

QUESTION 1

An insurance broker signed contracts with 15 people. The monthly premium (in rands) payable on each contract is given below.

134	215	325	326	362	429	515	531	598	610	624	728	923	1 034	1 200

1.1 Calculate the mean of the data. (2)

1.2 Write down the standard deviation of the data. (1)

1.3 Calculate how many monthly premiums are within ONE standard deviation of the mean. (2)

1.4 The insurance company decided to increase the monthly premiums.

- Monthly premiums that were less than R500 increased by 18%
- Monthly premiums that were equal to or more than R500 increased by k %

After these increases were applied to the above data, the new mean monthly premium was R686,44. Calculate the value of k. (4)
[9]

QUESTION 2

The manager of a supermarket decided to do a survey on the number of items that a customer ordered online and the time (in minutes) that a packer took to have the order ready for delivery. The supermarket received 10 online orders on a certain day. The information for these 10 orders is shown in the table below.

Number of items (x)	10	3	20	14	17	9	12	18	15	19
Time (in minutes) (y)	5	5	9	7	6	6	8	11	10	12

2.1 Draw a scatter plot on the grid provided in the ANSWER BOOK. (3)

2.2 Determine the equation of the least squares regression line. (3)

2.3 Write down the correlation coefficient of the data. (1)

2.4 The supermarket received an online order for 13 items. Predict how long (in minutes) it will take a packer to pack the order and have it ready for delivery. (2)

2.5 Explain why the y-intercept of the least squares regression line in QUESTION 2.2 does NOT make sense in this context. (1)
[10]

Paper J

QUESTION 1

1.1 The owner of a small company wishes to establish whether advertising in a regional newspaper is effective. The table below shows the amount spent on advertising and the corresponding sales figures for the last 9 years.

Amount spent on advertising (in rands) (x)	21 300	23 700	24 800	30 540	24 100	40 680	22 400	35 250	29 110
Sales (in rands) (y)	311 500	326 700	349 200	470 000	316 100	564 200	314 000	487 300	392 900

1.1.1 Determine the equation of the least squares regression line for the data. (3)

1.1.2 Predict the sales for a year in which the company will spend R28 500 on advertising. (2)

1.1.3 Write down the correlation coefficient of the data. (1)

1.1.4 Describe the association between the amount spent on advertising in the regional newspaper and the sales of this company. (1)

1.2 The profit that the small company made over the same 9 years is given in the table below.

Profit (in rands)	110 750	107 376	152 338	244 480	144 021	275 994	121 900	207 636	187 700

1.2.1 Calculate the mean profit made over the 9 years. (2)

1.2.2 Write down the standard deviation for the data. (1)

1.2.3 Determine the number of years in which the company made a profit that was greater than one standard deviation above the mean. (2)

[12]

QUESTION 2

The ages of the people who attended a music concert was summarised in the table below.

AGE	NUMBER OF PEOPLE
$5 < x \leq 15$	20
$15 < x \leq 25$	25
$25 < x \leq 35$	60
$35 < x \leq 45$	90
$45 < x \leq 55$	55
$55 < x \leq 65$	40
$65 < x \leq 75$	30

2.1 Write down the modal class of the data. (1)

2.2 How many people attended the music concert? (1)

2.3 On the grid provided in the ANSWER BOOK, draw a cumulative frequency graph (ogive) to represent the above data. (4)

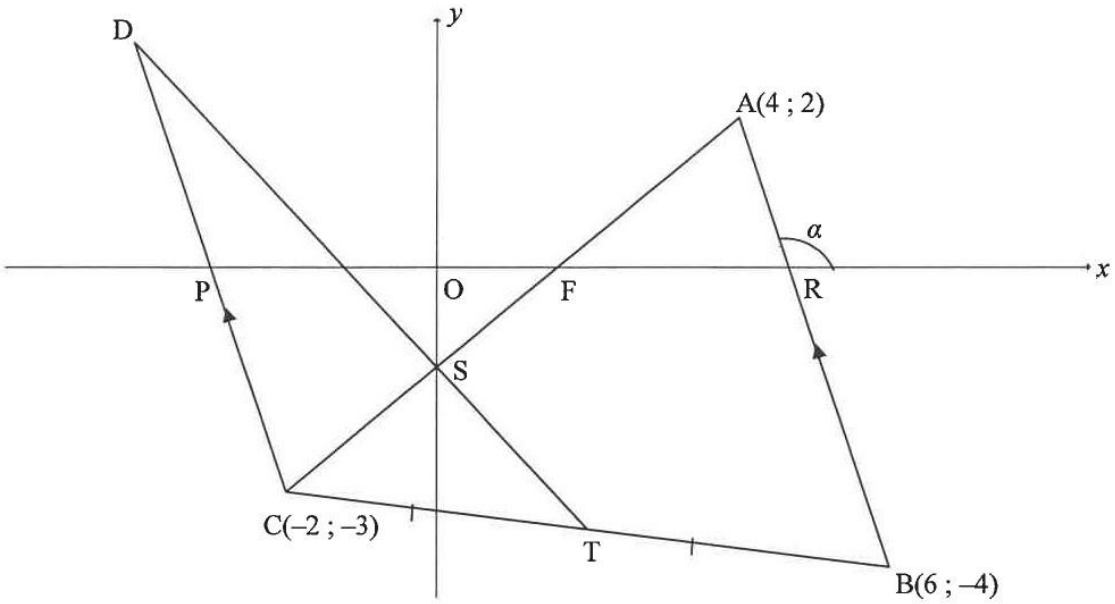
2.4 Use the cumulative frequency graph to determine the median age of the people who attended the music concert. (2)
[8]

ANALYTICAL GEOMETRY

Paper A

QUESTION 3

In the diagram, A(4 ; 2), B(6 ; –4) and C(–2 ; –3) are vertices of ΔABC. T is the midpoint of CB. The equation of line AC is $5x-6y=8$. The angle of inclination of AB is α. ΔDCT is drawn such that CD ∥ BA. The lines AC and DT intersect at S, the y-intercept of AC. P, F and R are the x-intercepts of DC, AC and AB respectively.



3.1 Calculate the:

3.1.1 Gradient of AB (2)

3.1.2 Size of α (2)

3.1.3 Coordinates of T (2)

3.1.4 Coordinates of S (2)

3.2 Determine the equation of CD in the form $y=mx+c$. (3)

3.3 Calculate the:

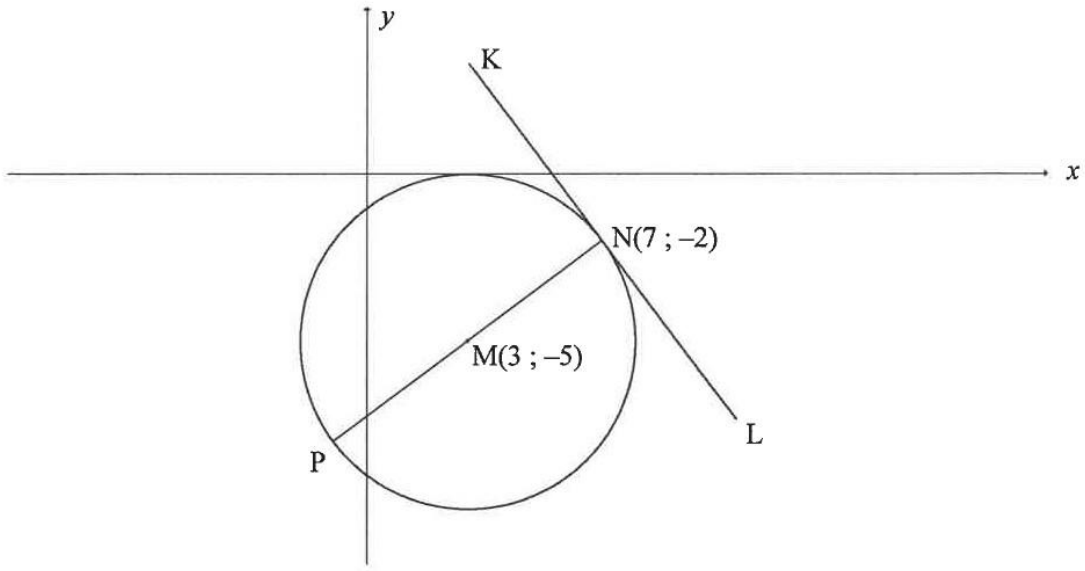
3.3.1 Size of $D\hat{C}A$ (4)

3.3.2 Area of POSC (5)

[20]

QUESTION 4

In the diagram, M(3 ; –5) is the centre of the circle having PN as its diameter. KL is a tangent to the circle at N(7 ; –2).



4.1 Calculate the coordinates of P. (2)

4.2 Determine the equation of:

4.2.1 The circle in the form $(x-a)^2+(y-b)^2=r^2$ (3)

4.2.2 KL in the form $y=mx+c$ (5)

4.3 For which values of k will $y=-\frac{4}{3}x+k$ be a secant to the circle? (4)

4.4 Points A(t ; t) and B are not shown on the diagram.

From point A, another tangent is drawn to touch the circle with centre M at B.

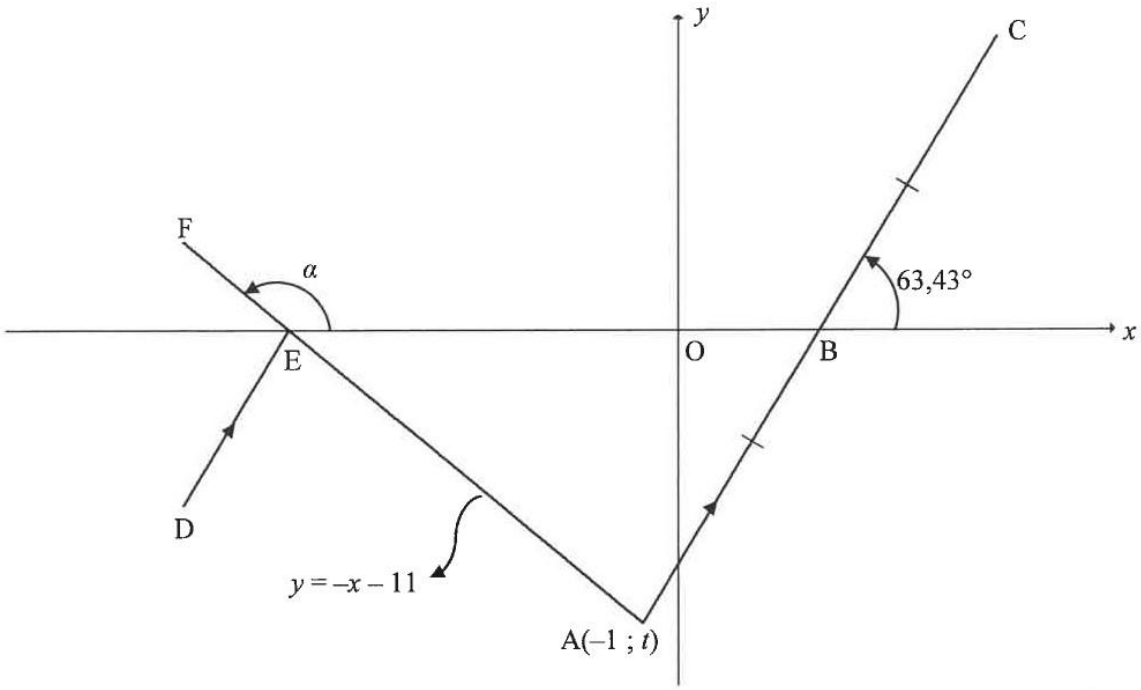
4.4.1 Show that the length of tangent AB is given by $\sqrt{2t^2+4t+9}$. (2)

4.4.2 Determine the minimum length of AB. (4)
[20]

Paper B

QUESTION 3

In the diagram, the equation of line AF is $y = -x - 11$. B, a point on the x-axis, is the midpoint of the straight line joining A$(-1 ; t)$ and C. The angles of inclination of AF and AC are α and 63,43° respectively. AF cuts the x-axis in E. D is a point such that DE ∥ AC.



3.1 Calculate the:

3.1.1 Value of t (2)

3.1.2 Size of α (2)

3.1.3 Gradient of AC, to the nearest whole number (2)

3.2 Determine the equation of AC in the form $y = mx + k$. (2)

3.3 Calculate the:

3.3.1 Coordinates of C (3)

3.3.2 Size of FÊD (3)

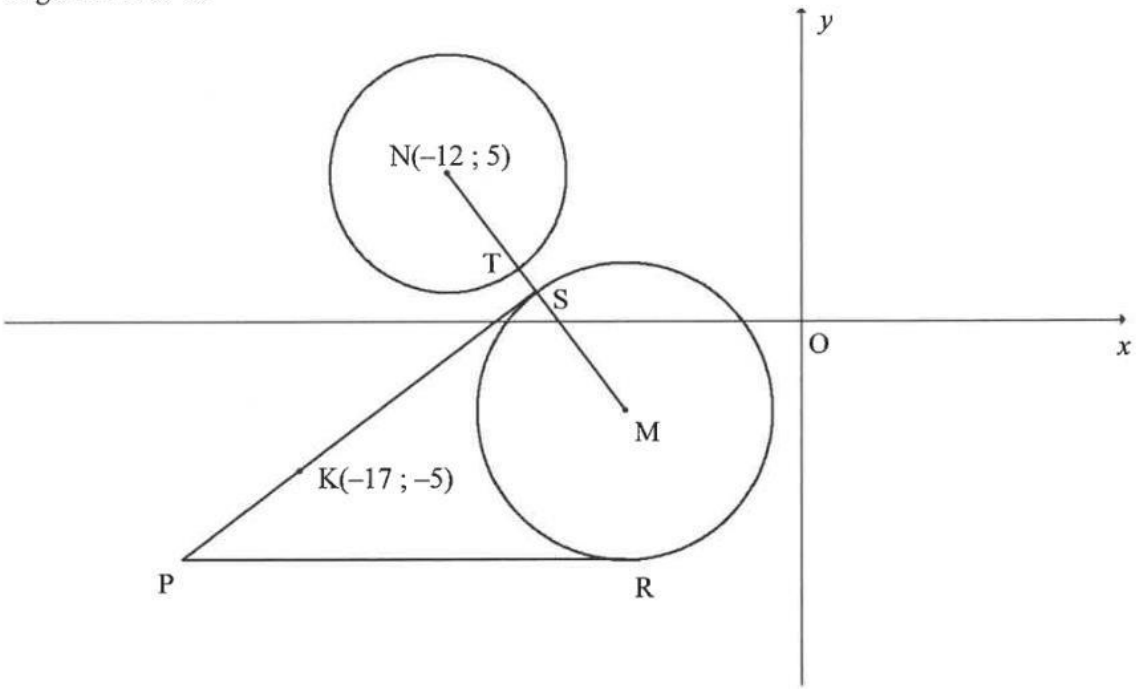
3.4 G is a point such that EAGC, in that order, is a parallelogram.

Determine the equation of a circle centred at G and passing through the point B.
Write your answer in the form $(x-a)^2 + (y-b)^2 = r^2$. (4)

[18]

QUESTION 4

In the diagram, the equation of the circle centred at N(–12 ; 5) is $x^2 + y^2 + 24x - 10y + 153 = 0$. The equation of the circle centred at M is $(x+6)^2 + (y+3)^2 = 25$. PS and PR are tangents to the circle centred at M at S and R respectively. PR is parallel to the x-axis. K(–17 ; –5) is a point on PS. The straight line joining N and M cuts the smaller circle at T and the larger circle at S.



4.1 Write down the coordinates of M. (2)

4.2 Calculate the:

4.2.1 Length of the radius of the smaller circle (2)

4.2.2 Length of TS (4)

4.3 Determine the equation of the tangent:

4.3.1 PR (2)

4.3.2 PS, in the form $y = mx + c$ (5)

4.4 Quadrilateral PSMR is drawn. Calculate the:

4.4.1 Perimeter of PSMR (5)

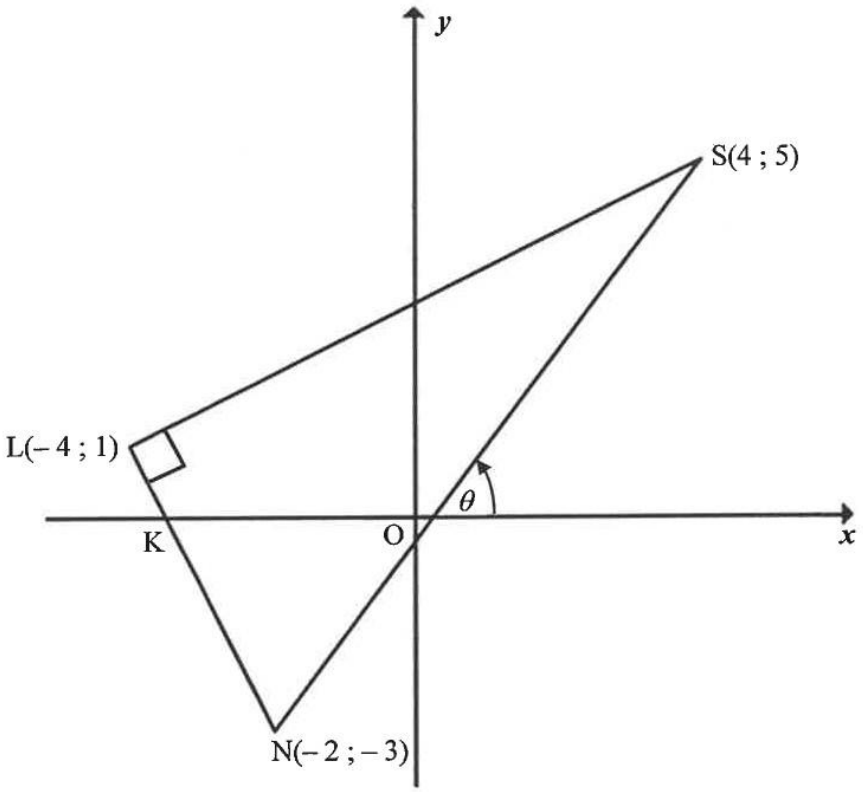
4.4.2 Ratio of $\dfrac{\text{area of } \Delta NPS}{\text{area of quadrilateral PSMR}}$ (2)

[22]

Paper C

QUESTION 3

In the figure, L(−4 ; 1), S(4 ; 5) and N(−2 ; −3) are the vertices of a triangle having $S\hat{L}N = 90°$. LN intersects the x-axis at K.



3.1 Calculate the length of SL. Leave your answer in surd form. (2)

3.2 Calculate the gradient of SN. (2)

3.3 Calculate the size of θ, the angle of inclination of SN. (2)

3.4 Calculate the size of $L\hat{N}S$. (3)

3.5 Determine the equation of the line which passes through L and is parallel to SN. Write your answer in the form $y = mx + c$. (3)

3.6 Calculate the area of ΔLSN. (3)

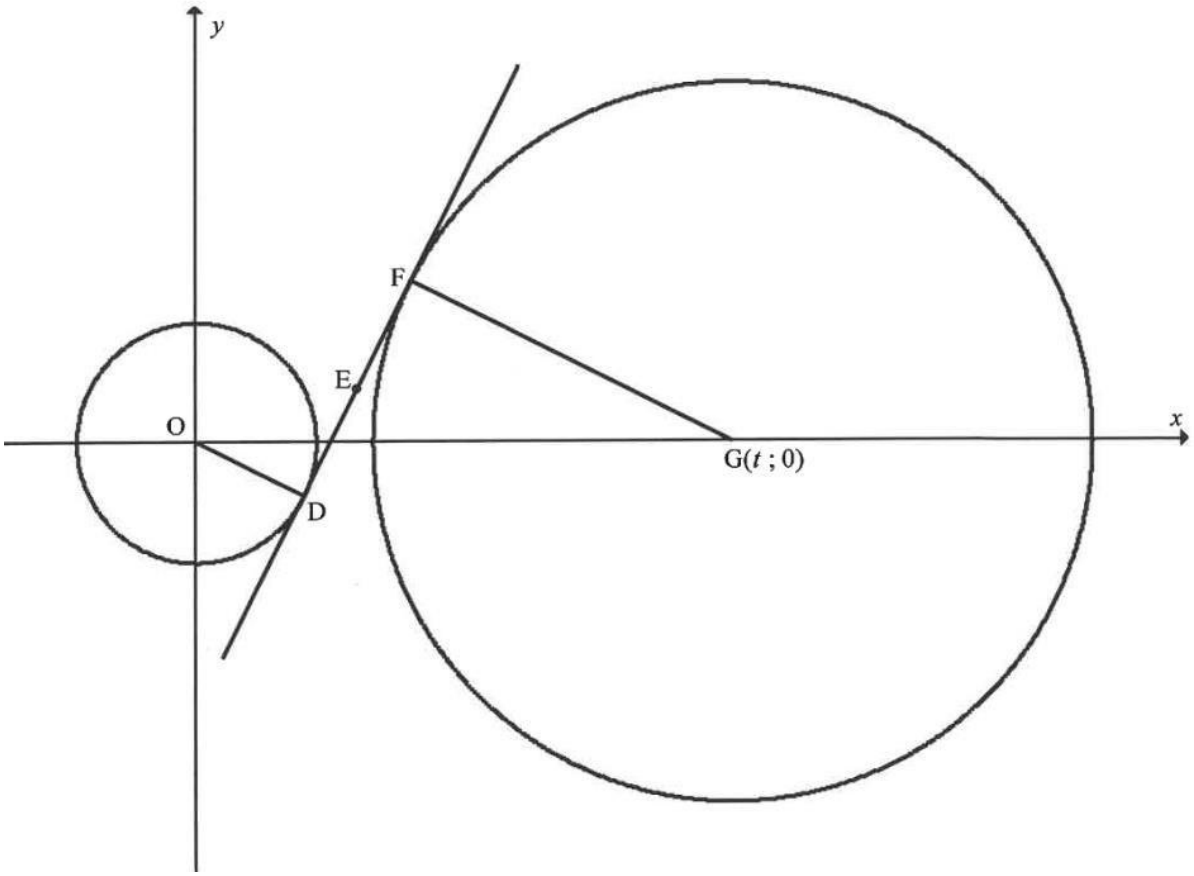
3.7 Calculate the coordinates of point P, which is equidistant from L, S and N. (3)

3.8 Calculate the size of $L\hat{P}S$. (2)

[20]

QUESTION 4

In the diagram, the circle with centre O has the equation $x^2 + y^2 = 20$. G$(t\ ;\ 0)$ is the centre of the larger circle. A common tangent touches the circles at D and F respectively, such that D$(p\ ;\ -2)$ lies in the 4th quadrant.



4.1 Given that D$(p\ ;\ -2)$ lies on the smaller circle, show that $p = 4$. (2)

4.2 E$(6\ ;\ 2)$ is the midpoint of DF. Determine the coordinates of F. (3)

4.3 Determine the equation of the common tangent, DF, in the form $y = mx + c$. (4)

4.4 Calculate the value of t. Show ALL working. (3)

4.5 Determine the equation of the larger circle in the form $ax^2 + by^2 + cx + dy + e = 0$. (4)

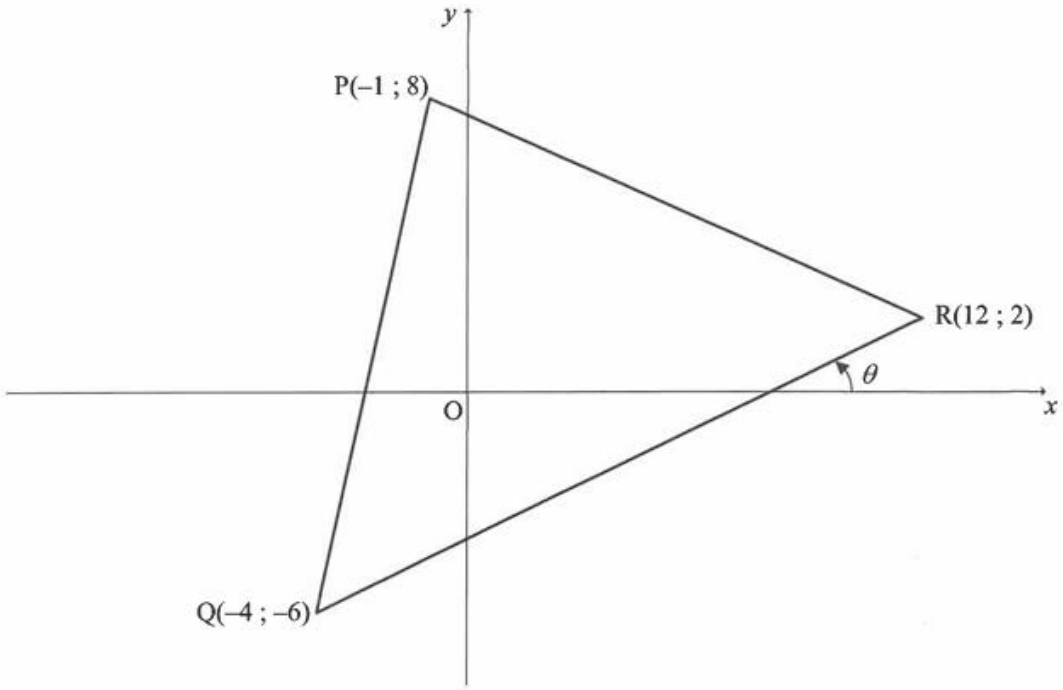
4.6 The smaller circle must be translated by k units along the x-axis to touch the larger circle internally. Calculate the possible values of k. (4)

[20]

Paper D

QUESTION 3

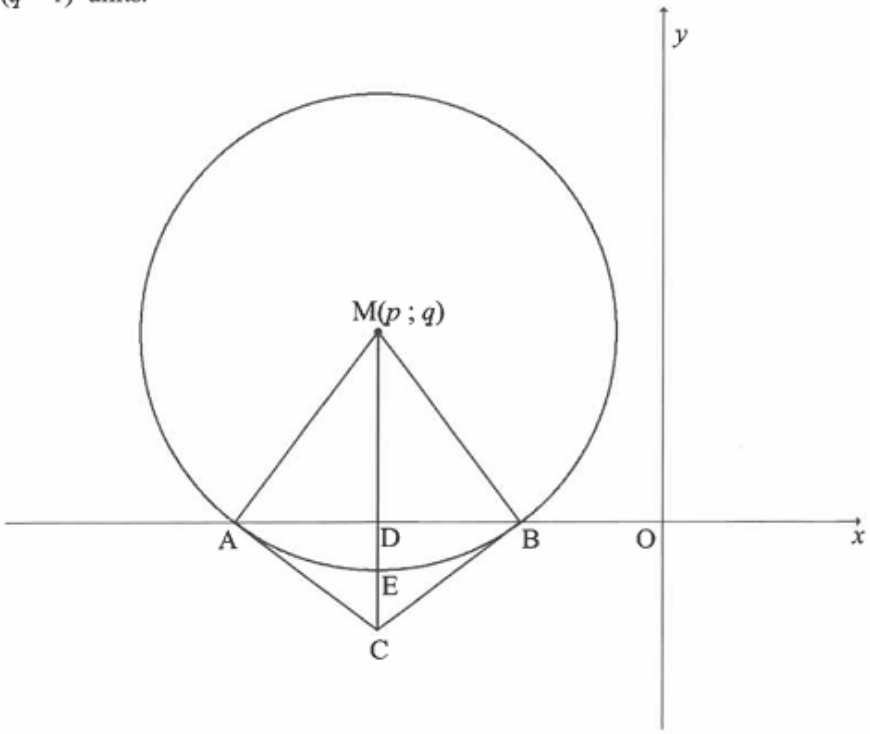
In the diagram, P(–1 ; 8), Q(–4 ; –6) and R(12 ; 2) are the vertices of ΔPQR. The angle of inclination of QR is θ.



3.1	Calculate the length of QR. Leave your answer in simplified surd form.	(2)
3.2	Calculate the gradient of QR.	(2)
3.3	Calculate the size of θ.	(2)
3.4	Determine the equation of QR.	(2)
3.5	PQRS, in that order, is a parallelogram. Write down the coordinates of S.	(2)
3.6	T is a point on QR such that PT ⊥ QR. Calculate the coordinates of T.	(5)
3.7	Calculate the area of parallelogram PQRS.	(3)
		[18]

QUESTION 4

In the diagram, M(p ; q) is the centre of the circle that intersects the x-axis at A and B. C is a point such that the line drawn from M to C is parallel to the y-axis and intersects the x-axis at D. MC intersects the circle at E(–6 ; –1). Tangents drawn from C touch the circle at A and B. AD = (q – 1) units.



4.1 Write down the value of p. (1)

4.2 Show that $q = 4$. (4)

4.3 Determine the equation of the circle in the form $(x-a)^2+(y-b)^2=r^2$. (2)

4.4 If the circle is translated 2 units to the left, determine the minimum distance between the circle and the y-axis. (1)

4.5 Calculate the coordinates of A and B. (3)

4.6 Determine the equation of tangent BC. (4)

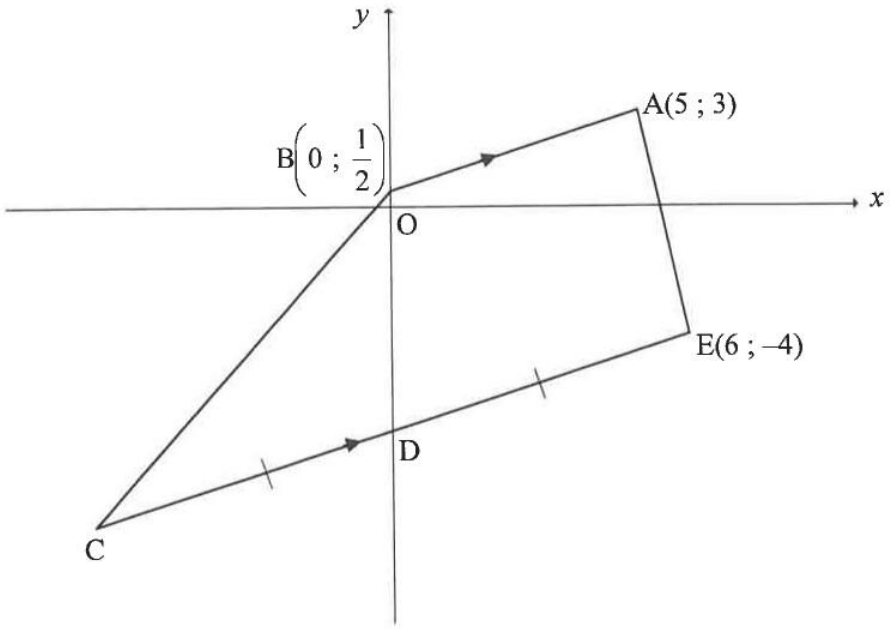
4.7 Write down the coordinates of C. (2)

4.8 Calculate the size of $A\hat{C}B$. (4)
[21]

Paper E

QUESTION 3

In the diagram, A(5 ; 3), $B\left(0 ; \frac{1}{2}\right)$, C and E(6 ; –4) are the vertices of a trapezium having BA || CE. D is the y-intercept of CE and CD = DE.



3.1 Calculate the gradient of AB. (2)

3.2 Determine the equation of CE in the form $y = mx + c$. (3)

3.3 Calculate the:

3.3.1 Coordinates of C (3)

3.3.2 Area of quadrilateral ABCD (4)

3.4 If point K is the reflection of E in the y-axis:

3.4.1 Write down the coordinates of K (2)

3.4.2 Calculate the:

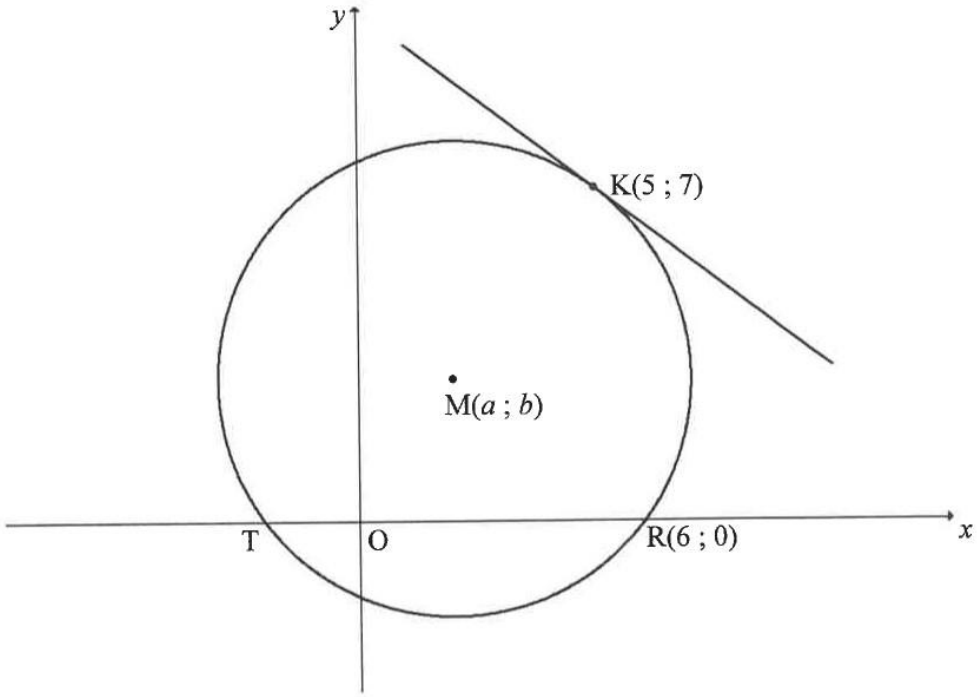
(a) Perimeter of ΔKEC (4)

(b) Size of $K\hat{C}E$ (3)

[21]

QUESTION 4

In the diagram, the circle centred at M(a ; b) is drawn. T and R(6 ; 0) are the x-intercepts of the circle. A tangent is drawn to the circle at K(5 ; 7).



4.1 M is a point on the line $y = x + 1$.

4.1.1 Write b in terms of a. (1)

4.1.2 Calculate the coordinates of M. (5)

4.2 If the coordinates of M are (2 ; 3), calculate the length of:

4.2.1 The radius of the circle (2)

4.2.2 TR (2)

4.3 Determine the equation of the tangent to the circle at K. Write your answer in the form $y = mx + c$. (5)

4.4 A horizontal line is drawn as a tangent to the circle M at the point N(c ; d), where $d < 0$.

4.4.1 Write down the coordinates of N. (2)

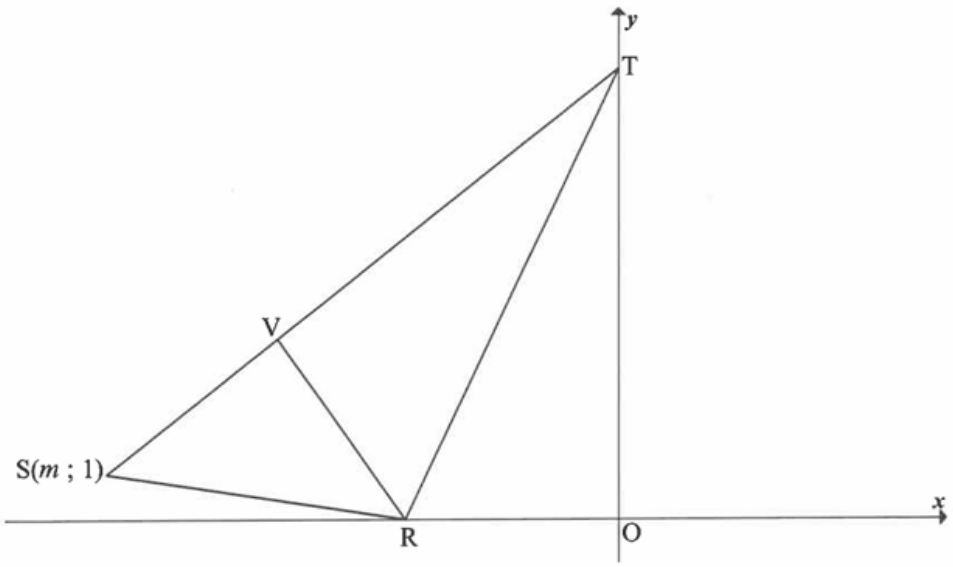
4.4.2 Determine the equation of the circle centred at N and passing through T. Write your answer in the form $(x-a)^2 + (y-b)^2 = r^2$. (3)

[20]

Paper F

QUESTION 3

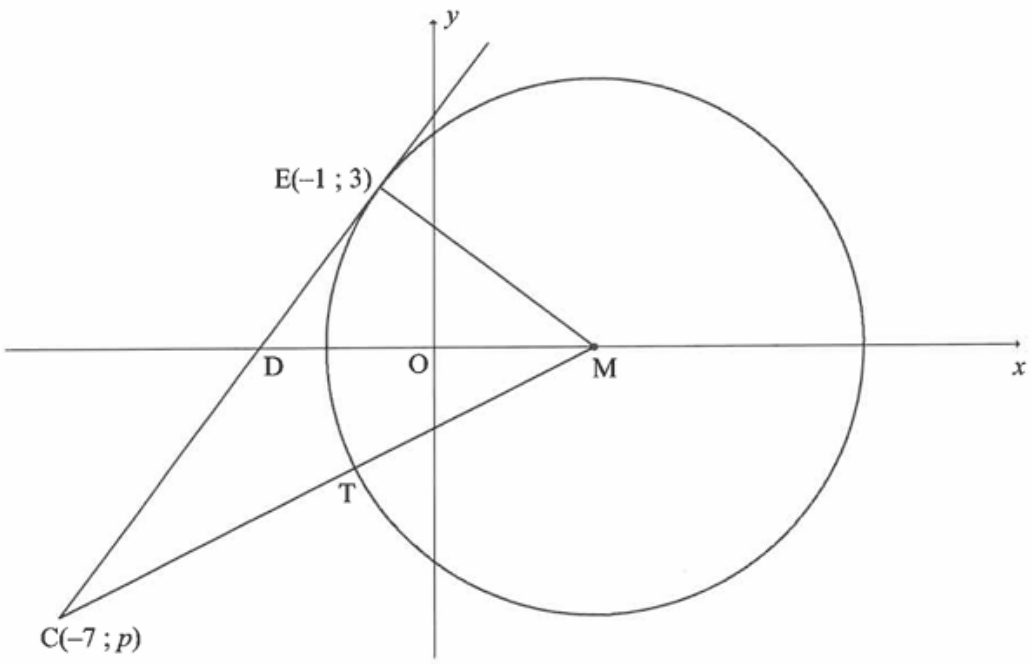
In the diagram below, ΔSRT is drawn where R lies on the x-axis and S lies to the left of R. T lies on the y-axis and the coordinates of S are $(m\ ;\ 1)$. The equation of RT is $2x-y+10=0$.



3.1	Calculate the coordinates of R.	(2)
3.2	Calculate the length of RT. Leave your answer in surd form.	(3)
3.3	If it is also given that $2RT^2 = 5SR^2$, calculate the value of m.	(4)
3.4	It is further given that V lies on ST such that VR is perpendicular to ST. Determine the equation of VR in the form $y = mx + c$.	(5)
3.5	Hence, show that the coordinates of V are $(-8\ ;\ 4)$.	(2)
3.6	If R′ is the reflection of R about the line $x = 0$, calculate the area of RVTR′.	(5)
		[21]

QUESTION 4

In the diagram, M is the centre of the circle having equation $(x-3)^2+y^2=25$. E(–1 ; 3) and T are points on the circle. EC is a tangent to the circle at E and cuts the x-axis at D. $ED=\frac{15}{4}$ units. MT is produced to meet the tangent at C(–7 ; p).



4.1 Write down the size of $C\hat{E}M$. (1)

4.2 Determine the equation of the tangent EC in the form $y=mx+c$. (4)

4.3 Calculate the length of DM. (3)

4.4 Show that $p=-5$ (1)

4.5 Calculate the coordinates of S if SEMC is a parallelogram and $x_s<0$. (3)

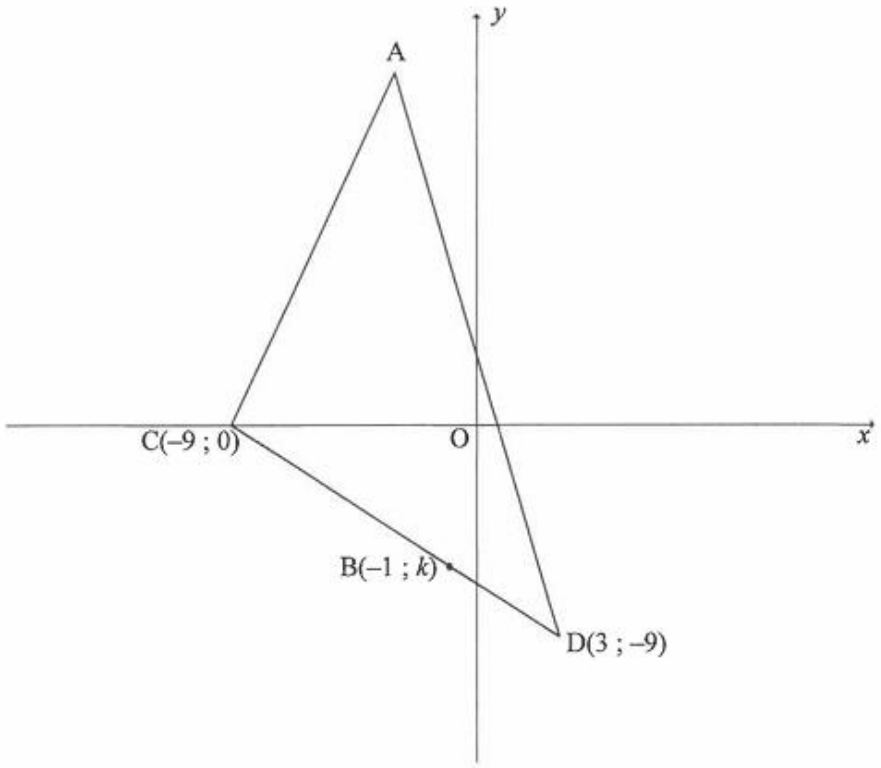
4.6 If the radius of the circle, centred at M, is increased by 7 units, determine whether S lies inside or outside the new circle. Support your answer with the necessary calculations. (3)

4.7 If ET is drawn, calculate the size of $E\hat{T}M$. (5)
[20]

Paper G

QUESTION 3

In the diagram below, ΔACD has vertices A, D(3 ; –9) and C(–9 ; 0), where A is a point in the second quadrant. B(–1 ; k) lies on side DC.



3.1 Calculate the gradient of DC. (2)

3.2 Determine the equation of DC in the form $y = mx + c$. (2)

3.3 Show that $k = -6$. (1)

3.4 Calculate the length of DC. (2)

3.5 Calculate the ratio of $\frac{DB}{DC}$. (2)

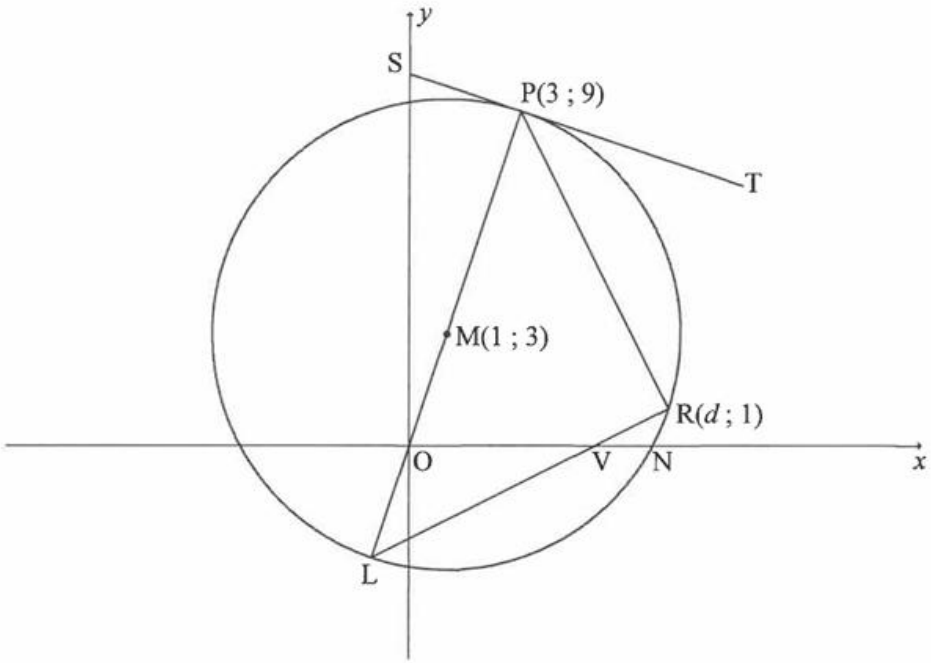
3.6 If M is a point on AD such that AC ∥ MB, calculate the ratio of $\frac{\text{Area } \Delta MBD}{\text{Area } \Delta ACD}$. (4)

3.7 If it is further given that the gradient of AD is -4 and the length of AD is $\sqrt{612}$ units, calculate the coordinates of A. (6)

[19]

QUESTION 4

In the diagram, M(1 ; 3) is the centre of the circle. The circle cuts the x-axis at N. ST is a tangent to the circle at P(3 ; 9). R(d ; 1), with $d > 0$, and L lie on the circle. O and V are the x-intercepts of PL and RL respectively.



4.1 Write down the coordinates of L. (2)

4.2 Determine the equation of tangent ST to the circle at P. (4)

4.3 Show that the equation of the circle with centre M is $x^2 + y^2 - 2x - 6y - 30 = 0$. (4)

4.4 Show that $d = 7$. (2)

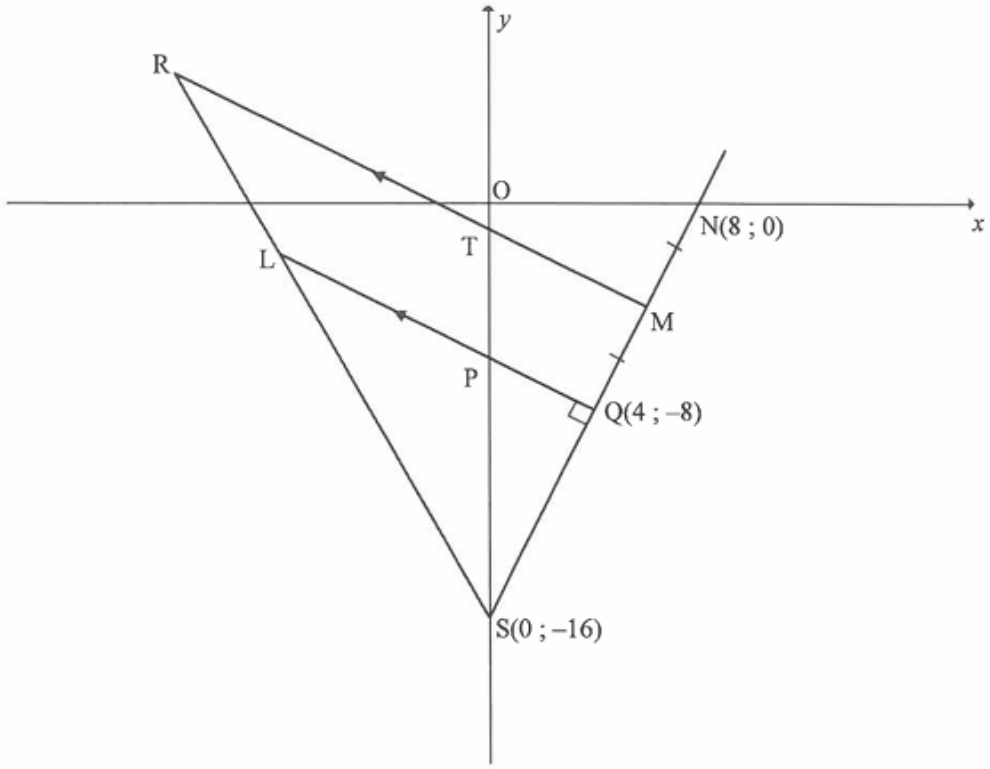
4.5 Calculate the size of $\hat{L}$ (5)

4.6 TR is a tangent to the circle at R. Prove that PT $\perp$ RT. (3)
[20]

Paper H

QUESTION 3

In the diagram, S(0 ; –16), L and Q(4 ; –8) are the vertices of ΔSLQ having LQ perpendicular to SQ. SL and SQ are produced to points R and M respectively such that RM ∥ LQ. SM produced cuts the x-axis at N(8 ; 0). QM = MN. T and P are the y-intercepts of RM and LQ respectively.



3.1 Calculate the coordinates of M. (2)

3.2 Calculate the gradient of NS. (2)

3.3 Show that the equation of line LQ is $y = -\frac{1}{2}x - 6$. (3)

3.4 Determine the equation of a circle having centre at O, the origin, and also passing through S. (2)

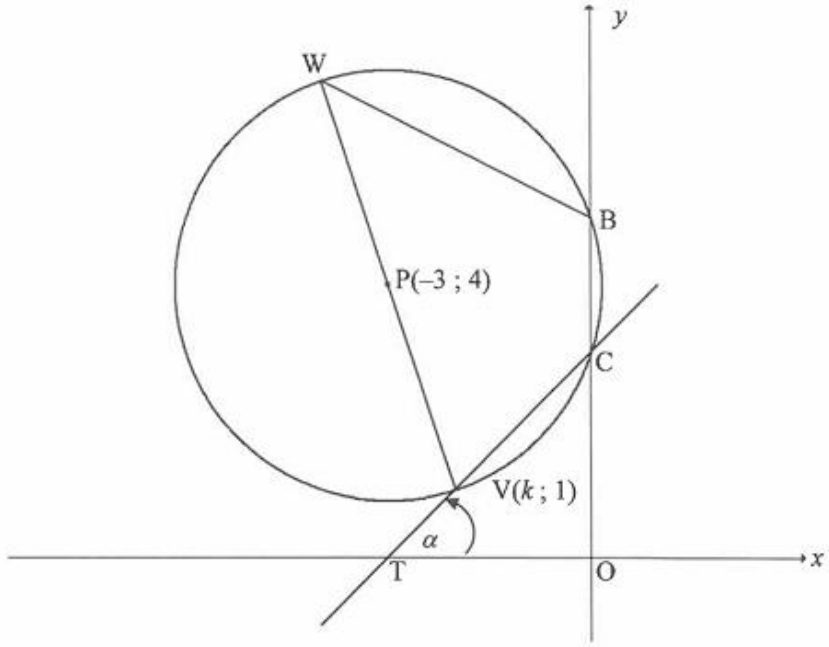
3.5 Calculate the coordinates of T. (3)

3.6 Determine $\frac{LS}{RS}$. (3)

3.7 Calculate the area of PTMQ. (4)
[19]

QUESTION 4

In the diagram, P(–3 ; 4) is the centre of the circle. V(k ; 1) and W are the endpoints of a diameter. The circle intersects the y-axis at B and C. BCVW is a cyclic quadrilateral. CV is produced to intersect the x-axis at T. $O\hat{T}C = \alpha$.



4.1 The radius of the circle is $\sqrt{10}$. Calculate the value of k if point V is to the right of point P. Clearly show ALL calculations. (5)

4.2 The equation of the circle is given as $x^2 + 6x + y^2 - 8y + 15 = 0$. Calculate the length of BC. (4)

4.3 If $k = -2$, calculate the size of:

4.3.1 α (3)

4.3.2 $V\hat{W}B$ (2)

4.4 A new circle is obtained when the given circle is reflected about the line $y = 1$.

Determine the:

4.4.1 Coordinates of Q, the centre of the new circle (2)

4.4.2 Equation of the new circle in the form $(x-a)^2 + (y-b)^2 = r^2$ (2)

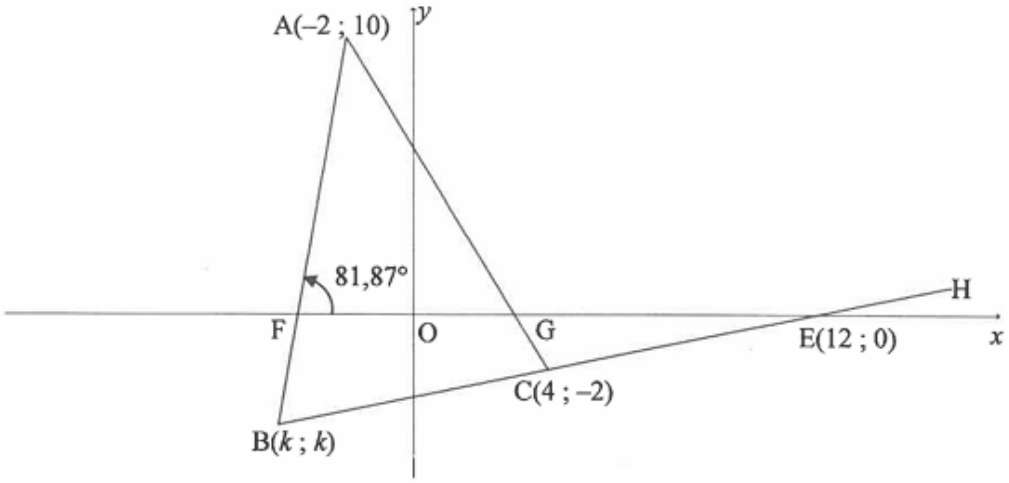
4.4.3 Equations of the lines drawn parallel to the y-axis and passing through the points of intersection of the two circles (2)

[20]

Paper I

QUESTION 3

In the diagram, A(–2 ; 10), B(k ; k) and C(4 ; –2) are the vertices of ΔABC. Line BC is produced to H and cuts the x-axis at E(12 ; 0). AB and AC intersects the x-axis at F and G respectively. The angle of inclination of line AB is 81,87°.



3.1 Calculate the gradient of:

3.1.1 BE (2)

3.1.2 AB (2)

3.2 Determine the equation of BE in the form $y = mx + c$ (2)

3.3 Calculate the:

3.3.1 Coordinates of B, where $k < 0$ (2)

3.3.2 Size of $\hat{A}$ (4)

3.3.3 Coordinates of the point of intersection of the diagonals of parallelogram ACES, where S is a point in the first quadrant (2)

3.4 Another point T(p ; p), where $p > 0$, is plotted such that ET = BE = $4\sqrt{17}$ units.

3.4.1 Calculate the coordinates of T. (5)

3.4.2 Determine the equation of the:

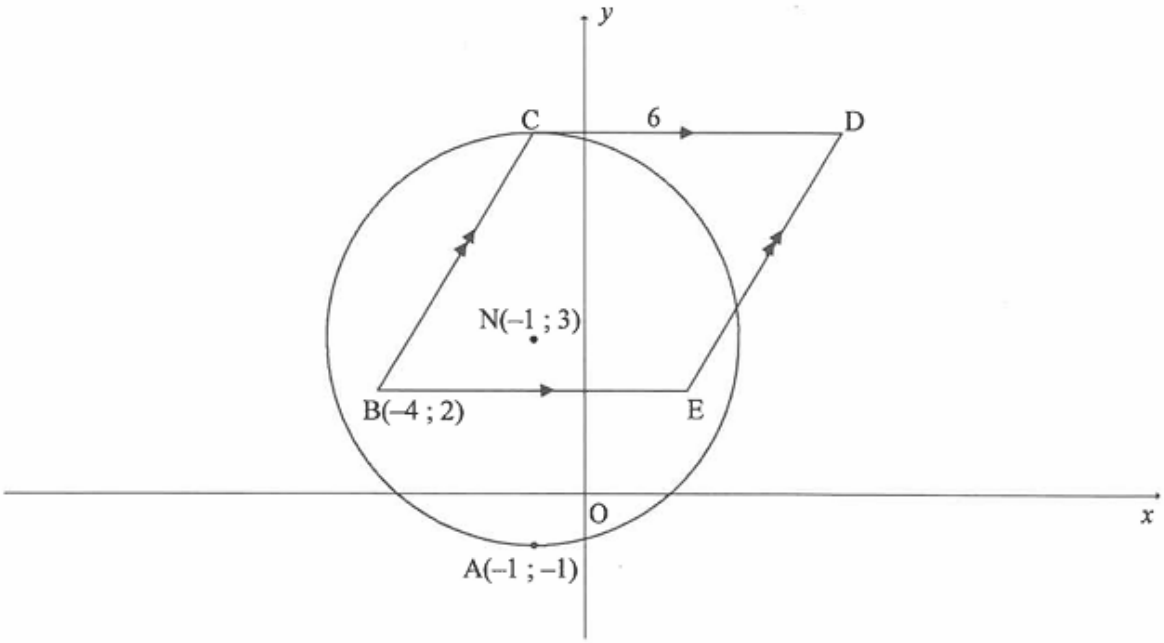
(a) Circle with centre at E and passing through B and T in the form $(x-a)^2 + (y-b)^2 = r^2$ (2)

(b) Tangent to the circle at point B(k ; k) (3)

[24]

QUESTION 4

In the diagram, the circle centred at N(–1 ; 3) passes through A(–1 ; –1) and C. B(–4 ; 2), C, D and E are joined to form a parallelogram such that BE is parallel to the x-axis. CD is a tangent to the circle at C and CD = 6 units.



4.1 Write down the length of the radius of the circle. (1)

4.2 Calculate the:

4.2.1 Coordinates of C (2)

4.2.2 Coordinates of D (2)

4.2.3 Area of ΔBCD (3)

4.3 The circle, centred at N, is reflected about the line $y = x$. M is the centre of the new circle which is formed. The two circles intersect at A and F.

Calculate the:

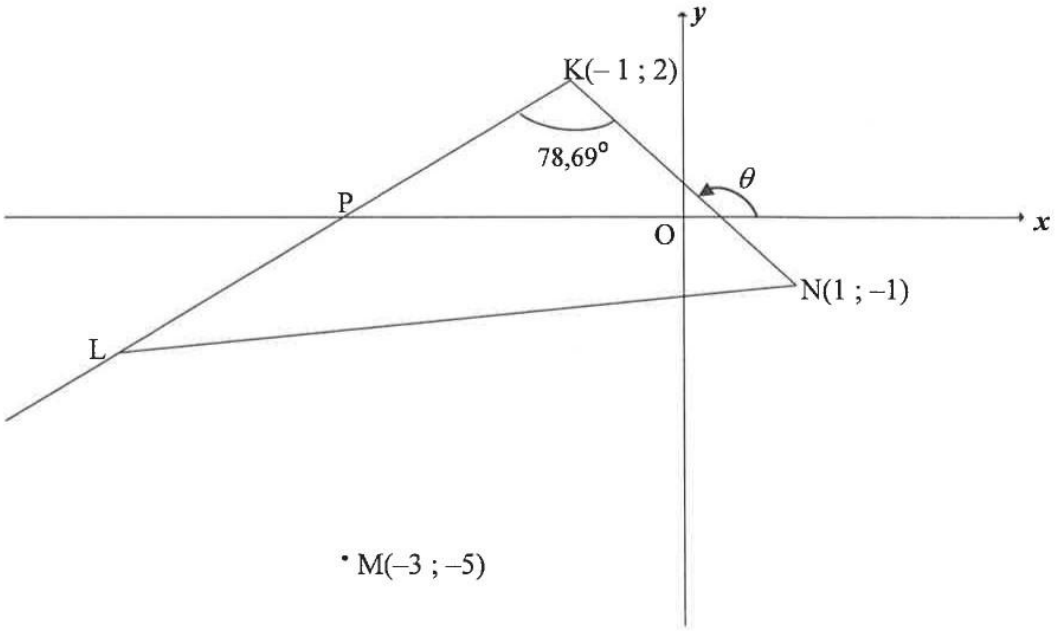
4.3.1 Length of NM (3)

4.3.2 Midpoint of AF (4)
[15]

Paper J

QUESTION 3

In the diagram, K(– 1 ; 2), L and N(1 ; – 1) are vertices of ΔKLN such that $L\hat{K}N = 78{,}69°$. KL intersects the x-axis at P. KL is produced. The inclination of KN is θ. The coordinates of M are (–3 ; –5).



3.1 Calculate:

3.1.1 The gradient of KN (2)

3.1.2 The size of θ, the inclination of KN (2)

3.2 Show that the gradient of KL is equal to 1. (2)

3.3 Determine the equation of the straight line KL in the form $y = mx + c$. (2)

3.4 Calculate the length of KN. (2)

3.5 It is further given that KN = LM.

3.5.1 Calculate the possible coordinates of L. (5)

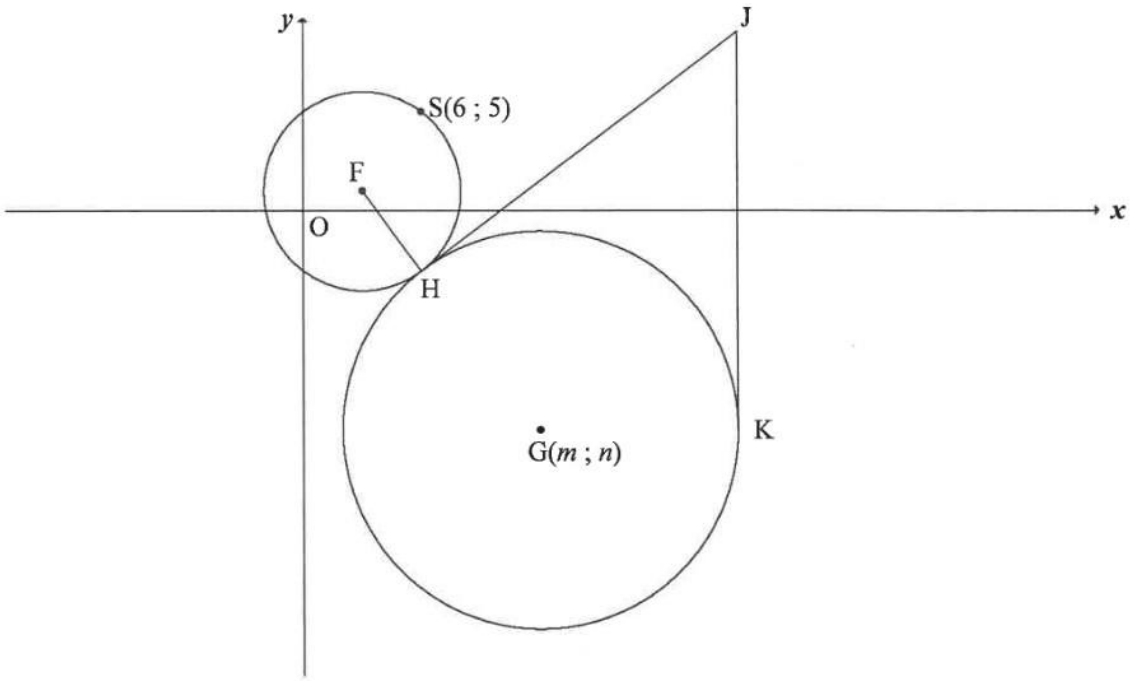
3.5.2 Determine the coordinates of L if it is given that KLMN is a parallelogram. (3)

3.6 T is a point on KL produced. TM is drawn such that TM = LM. Calculate the area of ΔKTN. (4)

[22]

QUESTION 4

In the diagram, the equation of the circle with centre F is $(x-3)^2+(y-1)^2=r^2$. S(6 ; 5) is a point on the circle with centre F. Another circle with centre G$(m\,;n)$ in the 4th quadrant touches the circle with centre F, at H such that FH : HG = 1 : 2. The point J lies in the first quadrant such that HJ is a common tangent to both these circles. JK is a tangent to the larger circle at K.



4.1 Write down the coordinates of F. (2)

4.2 Calculate the length of FS. (2)

4.3 Write down the length of HG. (1)

4.4 Give a reason why JH = JK. (1)

4.5 Determine:

4.5.1 The distance FJ, with reasons, if it is given that JK = 20 (4)

4.5.2 The equation of the circle with centre G in terms of m and n in the form $(x-a)^2+(y-b)^2=r^2$ (1)

4.5.3 The coordinates of G, if it is further given that the equation of tangent JK is $x=22$ (7)

[18]

TRIGONOMETRY

Paper A

QUESTION 5

5.1 Given that $\sqrt{13}\sin x+3=0$, where $x\in(90°;270°)$.

Without using a calculator, determine the value of:

5.1.1 $\sin(360°+x)$ (2)

5.1.2 $\tan x$ (3)

5.1.3 $\cos(180°+x)$ (2)

5.2 Determine the value of the following expression, **without using a calculator**:

$$\frac{\cos(90°+\theta)}{\sin(\theta-180°)+3\sin(-\theta)}$$ (5)

5.3 Determine the general solution of the following equation:

$(\cos x+2\sin x)(3\sin 2x-1)=0$ (6)

5.4 Given the identity: $\cos(x+y).\cos(x-y)=1-\sin^2 x-\sin^2 y$

5.4.1 Prove the identity. (4)

5.4.2 Hence, determine the value of $1-\sin^2 45°-\sin^2 15°$, **without using a calculator**. (3)

5.5 Consider the trigonometric expression: $16\sin x.\cos^3 x-8\sin x.\cos x$

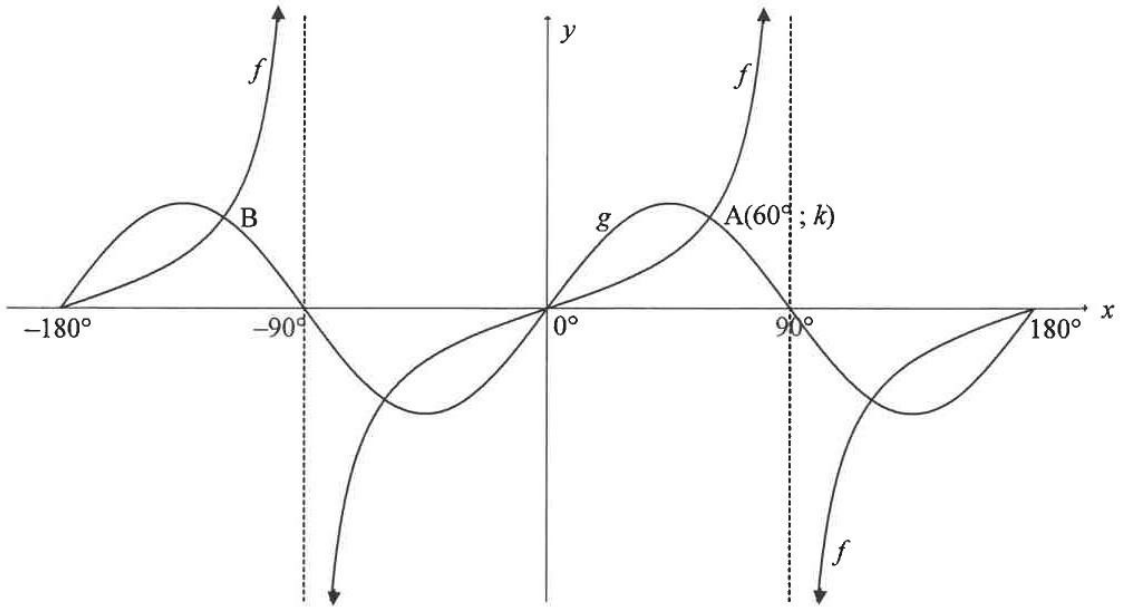
5.5.1 Rewrite the expression as a single trigonometric ratio. (4)

5.5.2 For which value of x in the interval $x\in[0°;90°]$ will $16\sin x.\cos^3 x-8\sin x.\cos x$ have its minimum value? (1)

[30]

QUESTION 6

In the diagram below, the graphs of $f(x)=\tan x$ and $g(x)=2\sin 2x$ are drawn for the interval $x\in[-180°;180°]$ A$(60°;k)$ and B are two points of intersection of f and g.



6.1 Write down the period of g. (1)

6.2 Calculate the:

6.2.1 Value of k (1)

6.2.2 Coordinates of B (1)

6.3 Write down the range of $2g(x)$. (2)

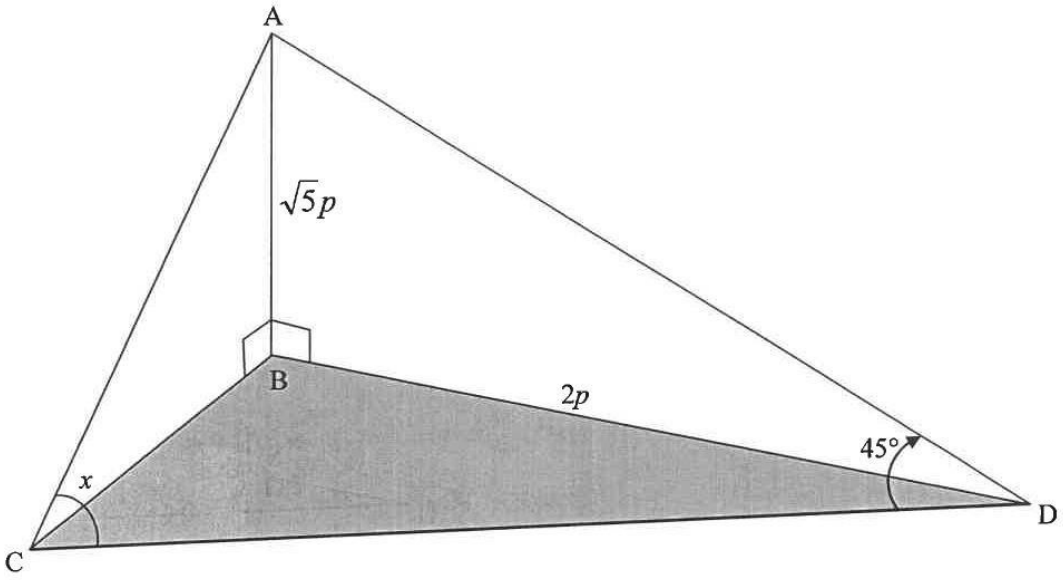
6.4 For which values of x will $g(x+5°)-f(x+5°)\leq 0$ in the interval $x\in[-90°;0°]$? (2)

6.5 Determine the values of p for which $\sin x.\cos x=p$ will have exactly two real roots in the interval $x\in[-180°;180°]$. (3)

[10]

QUESTION 7

AB is a vertical flagpole that is $\sqrt{5}p$ metres long. AC and AD are two cables anchoring the flagpole. B, C and D are in the same horizontal plane.
BD = $2p$ metres, $A\hat{C}D = x$ and $A\hat{D}C = 45°$.



7.1 Determine the length of AD in terms of p. (2)

7.2 Show that the length of $CD = \dfrac{3p(\sin x + \cos x)}{\sqrt{2}\sin x}$. (5)

7.3 If it is further given that $p = 10$ and $x = 110°$, calculate the area of ΔADC. (3)
[10]

Paper B

QUESTION 5

5.1 Simplify the expression to a **single trigonometric term:**

$$\tan(-x).\cos x.\sin(x-180°)-1$$ (5)

5.2 Given: $\cos 35° = m$

Without using a calculator, determine the value of EACH of the following in terms of m:

5.2.1 $\cos 215°$ (2)

5.2.2 $\sin 20°$ (3)

5.3 Determine the general solution of:

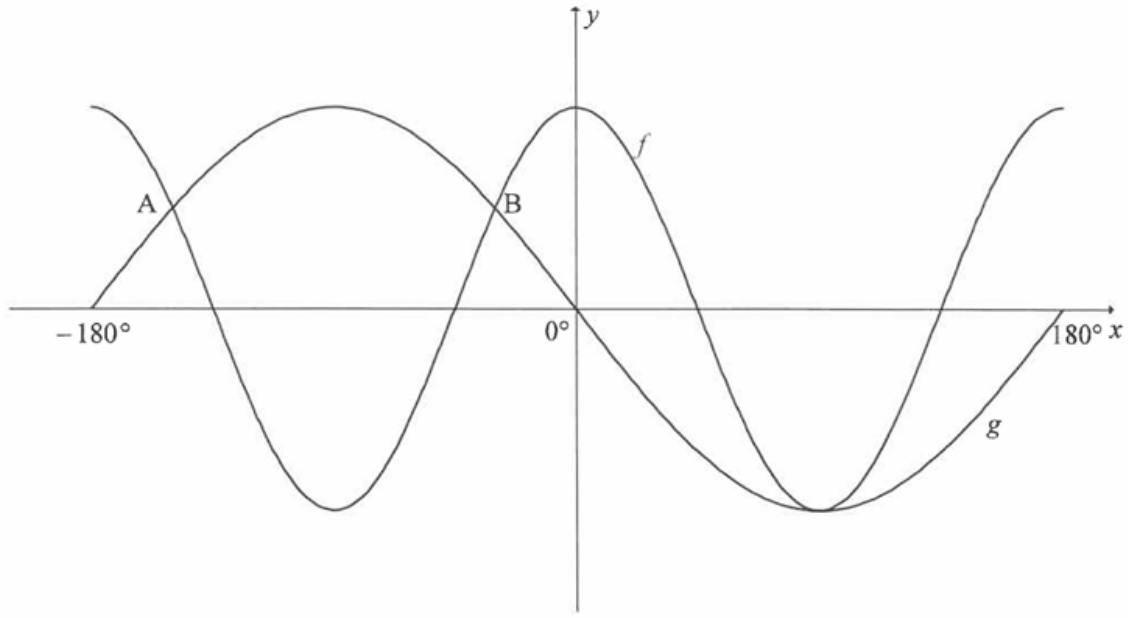
$$\cos 4x.\cos x + \sin x.\sin 4x = -0,7$$ (4)

5.4 Prove the identity: $\dfrac{\sin 4x.\cos 2x - 2\cos 4x.\sin x.\cos x}{\tan 2x} = \cos^2 x - \sin^2 x$ (4)

[18]

QUESTION 6

In the diagram below, the graphs of $f(x) = \cos 2x$ and $g(x) = -\sin x$ are drawn for the interval $x \in [-180° ; 180°]$ A and B are two points of intersection of f and g.



6.1 **Without using a calculator**, determine the values of x for which $\cos 2x = -\sin x$ in the interval $x \in [-180° ; 180°]$. (6)

6.2 Use the graphs above to answer the following questions:

6.2.1 How many degrees apart are points A and B from each other? (2)

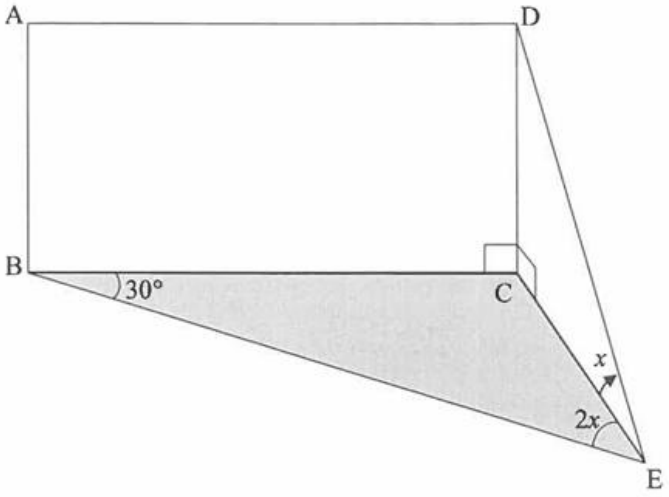
6.2.2 For which values of x in the given interval will $f'(x).g'(x) > 0$? (2)

6.2.3 Determine the values of k for which $\cos 2x + 3 = k$ will have no solution. (3)

[13]

QUESTION 7

Points B, C and E lie in the same horizontal plane. ABCD is a rectangular piece of board. CDE is a triangular piece of board having a right angle at C. Each piece of board is placed perpendicular to the horizontal plane and joined along DC, as shown in the diagram. The angle of elevation from E to D is x. $B\hat{E}C = 2x$ and $E\hat{B}C = 30°$.



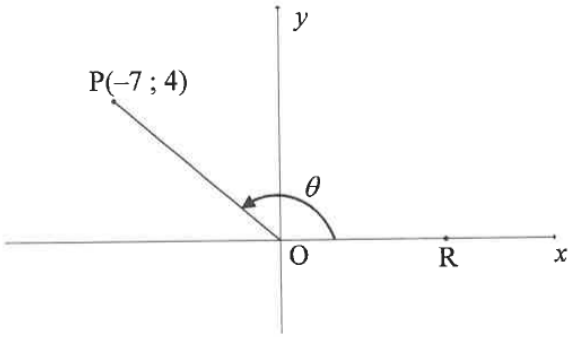
7.1 Show that $DC = \dfrac{BC}{4\cos^2 x}$ (6)

7.2 If $x = 30°$, show that the area of ABCD = $3AB^2$. (3)
[9]

Paper C

QUESTION 5

5.1 In the diagram below, P(–7 ; 4) is a point in the Cartesian plane. R is a point on the positive x-axis such that obtuse $\hat{POR} = \theta$.



Calculate, **without using a calculator**, the:

5.1.1 Length OP (2)

5.1.2 Value of:

(a) $\tan\theta$ (1)

(b) $\cos(\theta - 180°)$ (2)

5.2 Determine the general solution of: $\sin x\cos x + \sin x = 3\cos^2 x + 3\cos x$ (7)

5.3 Given the identity: $\dfrac{\sin 3x}{1-\cos 3x} = \dfrac{1+\cos 3x}{\sin 3x}$

5.3.1 Prove the identity given above. (3)

5.3.2 Determine the values of x, in the interval $x \in [0° ; 60°]$, for which the identity will be undefined. (3)

[18]

QUESTION 6

6.1 **Without using a calculator**, simplify the following expression to a single trigonometric term:

$$\frac{\sin 10^\circ}{\cos 440^\circ} + \tan(360^\circ - \theta).\sin 2\theta \qquad (6)$$

6.2 Given: $\sin(60^\circ + 2x) + \sin(60^\circ - 2x)$

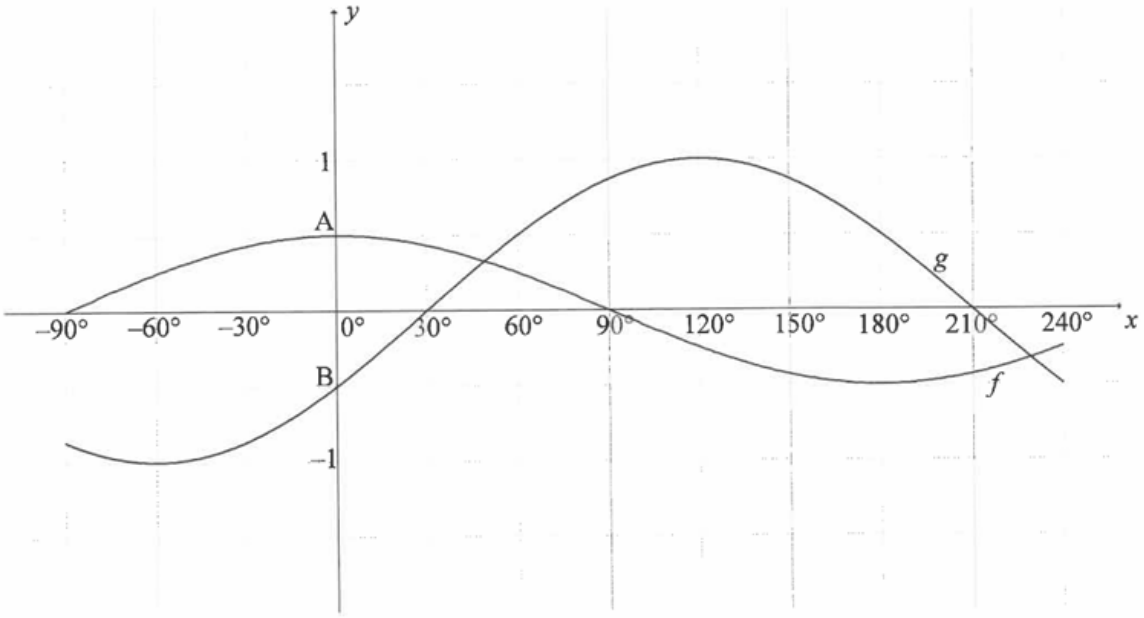
6.2.1 Calculate the value of k if $\sin(60^\circ + 2x) + \sin(60^\circ - 2x) = k\cos 2x$. (3)

6.2.2 If $\cos x = \sqrt{t}$, **without using a calculator**, determine the value of $\tan 60^\circ[\sin(60^\circ + 2x) + \sin(60^\circ - 2x)]$ in terms of t. (3)

[12]

QUESTION 7

In the diagram below, the graphs of $f(x)=\frac{1}{2}\cos x$ and $g(x)=\sin(x-30°)$ are drawn for the interval $x\in[-90°;240°]$. A and B are the y-intercepts of f and g respectively.



7.1 Determine the length of AB. (2)

7.2 Write down the range of $3f(x)+2$. (2)

7.3 Read off from the graphs a value of x for which $g(x)-f(x)=\frac{\sqrt{3}}{2}$. (2)

7.4 For which values of x, in the interval $x\in[-90°;240°]$, will:

7.4.1 $f(x).g(x)>0$ (2)

7.4.2 $g'(x-5°)>0$ (2)

[10]

QUESTION 8

FIGURE I shows a ramp leading to the entrance of a building. B, C and D lie on the same horizontal plane. The perpendicular height (AC) of the ramp is 0,5 m and the angle of elevation from B to A is 15°. The entrance of the building (AE) is 0,915 m wide.

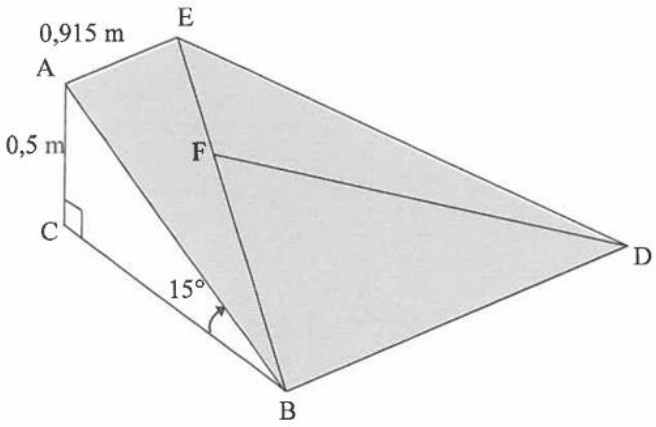


FIGURE I

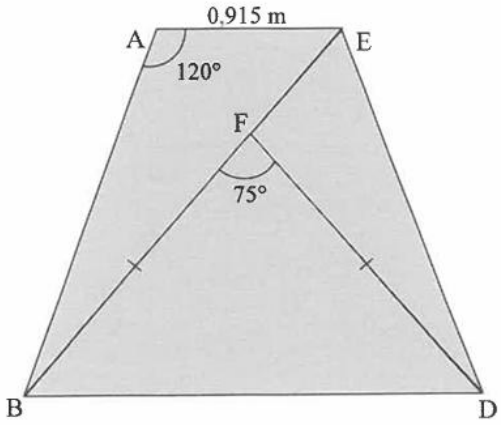


FIGURE II (top view)

8.1 Calculate the length of AB. (2)

8.2 Figure II shows the top view of the ramp. The area of the top of the ramp is divided into three triangles, as shown in the diagram.

If $B\hat{A}E = 120°$, calculate the length of BE. (3)

8.3 Calculate the area of ΔBFD if $B\hat{F}D = 75°$, $BF = FD$ and $BF = \frac{5}{7}BE$. (3)

[8]

Paper D

QUESTION 5

5.1 It is given that $\tan 50° = k$. Express EACH of the following in terms of k:

5.1.1 $\cos 40°$ (2)

5.1.2 $\dfrac{2\sin 25°.\cos 25°}{-2+4\sin^2 25°}$ (5)

5.1.3 $\sin 10°$ (4)

5.2 Given: $\dfrac{\sin(540°+x).\cos(90°+x)}{\sin(-x)}$

5.2.1 Simplify the expression above fully to a single trigonometric ratio. (4)

5.2.2 Hence, determine the values of x in the interval $x \in [0°;360°]$ for which $\sqrt{\dfrac{\sin(540°+x).\cos(90°+x)}{\sin(-x)}}$ will be real. (2)

[17]

QUESTION 6

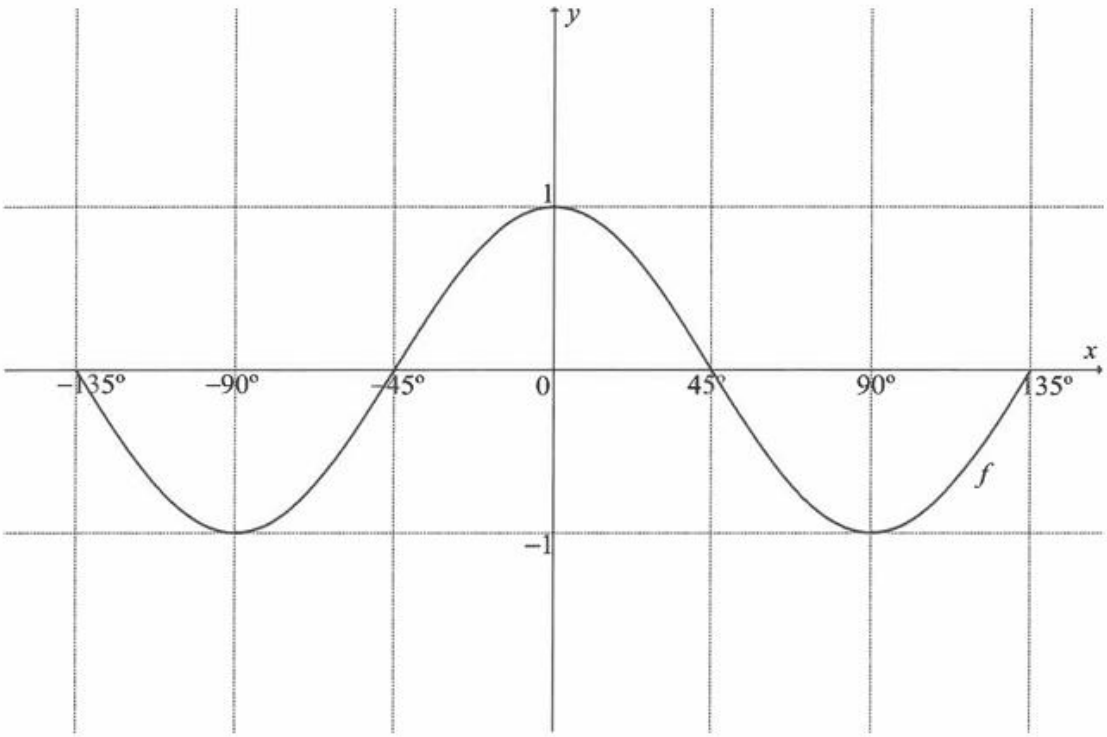
6.1 Prove that: $[\tan(180°-x)](1-\cos^2 x)+\cos^2 x = \dfrac{(\sin x-\cos x)(1+\sin x.\cos x)}{-\cos x}$ (6)

6.2 It is given that $\sin^2 x$; $\cos^2 x$ and $\dfrac{1}{2}\sin 2x$ are the first three terms of an arithmetic sequence. The constant difference of the arithmetic sequence is NOT zero.
Determine the general solution for x. (7)

[13]

QUESTION 7

In the diagram, the graph of $f(x)=\cos 2x$ is drawn for $x\in[-135°\,;135°]$.



7.1 Write down the period of f. (1)

7.2 On the set of axes provided in the ANSWER BOOK, draw the graph of $g(x)=\tan 2x-1$ for $x\in[-135°\,;135°]$. (3)

7.3 Graph f is translated $45°$ to the left to form graph h. Determine the equation of h in its simplest form. (1)

7.4 Write down the range of h. (1)

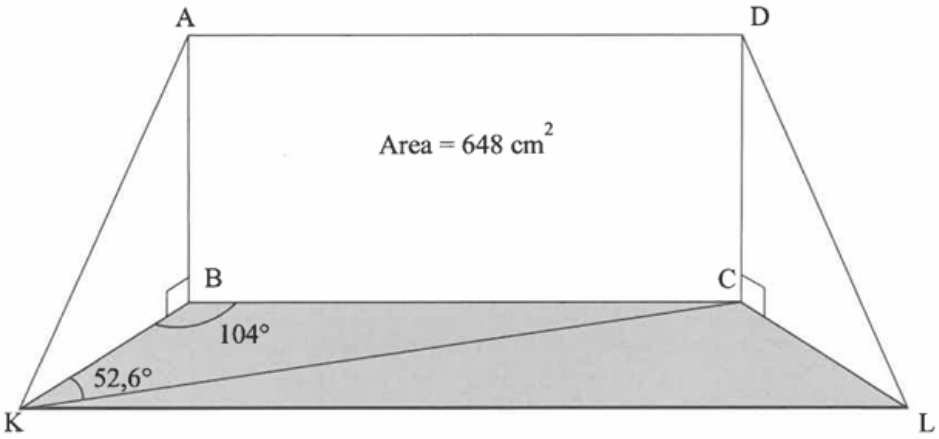
7.5 Determine the values of x for which $(1-\tan 2x)(\cos 2x)\geq 0$ in the interval $x\in[0°\,;135°]$. (4)

[10]

QUESTION 8

As part of a school project, learners are required to design a portable stage for a puppet show, as shown in the diagram below. The design must fulfil the following requirements:

- BKLC is a horizontal base having $K\hat{B}C = 104°$ and $B\hat{K}C = 52,6°$.
- The rectangular backdrop, ABCD, is vertical to the horizontal base and must have an area of 648 cm^2.
- The sides of ABCD must be in the ratio AB : BC = 1 : 2.
- The stage must be partly enclosed with triangular sides ABK and DCL.



8.1 Show that AB = 18 cm. (2)

8.2 Calculate the length of AC. (2)

8.3 Calculate the length of diagonal KC. (2)

8.4 If AB = BK, calculate the size of $K\hat{A}C$. (4)

[10]

Paper E

QUESTION 5

5.1 If $\cos\theta = -\frac{5}{13}$ where $180° < \theta < 360°$, determine, **without using a calculator**, the value of:

5.1.1 $\sin^2\theta$ (3)

5.1.2 $\tan(360° - \theta)$ (2)

5.1.3 $\cos(\theta - 135°)$ (4)

5.2 Simplify the expression to a single trigonometric term: $\frac{2\cos(180° - x)\sin(-x)}{1 - 2\cos^2(90° - x)}$ (6)

5.3 Calculate the value of the following expression **without using a calculator**: $(\tan 92°)(\tan 94°)(\tan 96°) \ldots (\tan 176°)(\tan 178°)$ (4)

[19]

QUESTION 6

6.1 Prove that $2\cos^2(45° + x) = 1 - \sin 2x$. (4)

6.2 Consider the expression: $\sin(A - B) - \sin(A + B)$

6.2.1 Prove that $\sin(A - B) - \sin(A + B) = -2\cos A \sin B$. (2)

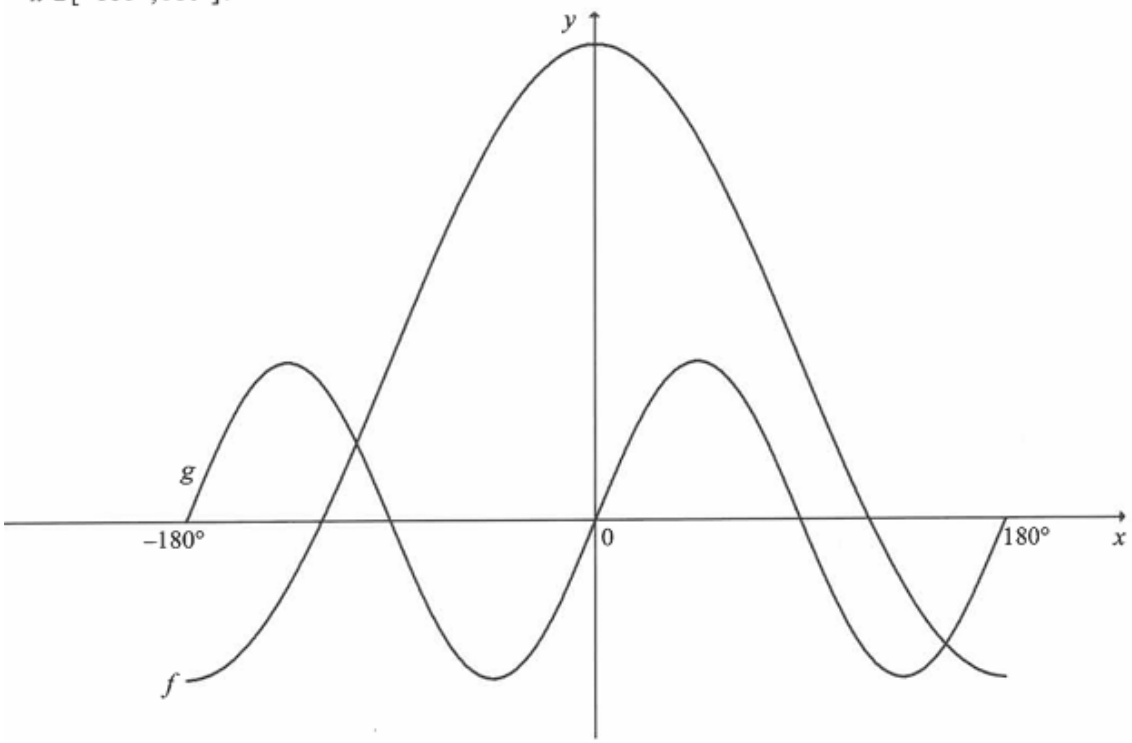
6.2.2 Simplify the following expression to a single term: $\sin 4x - \sin 10x$ (2)

6.2.3 Hence, determine the solution for $\sin 4x - \sin 10x = \sin 3x$ for $x \in [0°; 30°]$. (5)

[13]

QUESTION 7

In the diagram, the graphs of $f(x) = 2\cos x + 1$ and $g(x) = \sin 2x$ are drawn for the interval $x \in [-180°; 180°]$.



7.1 Write down the range of f. (1)

7.2 Write down the period of g. (1)

7.3 For which values of x, in the interval $x \in [-180°; 180°]$, is f increasing? (1)

7.4 Use the graphs to determine the values of x, in the interval $x \in [-180°; 180°]$, for which:

7.4.1 $g(x).f'(x) < 0$ (2)

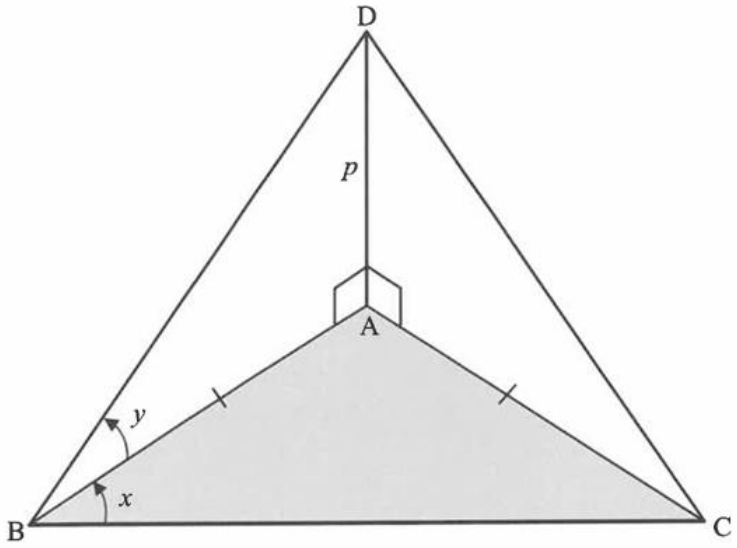
7.4.2 $\cos x \leq -\frac{1}{2}$ (3)

7.5 Graph g is shifted $45°$ to the right to obtain a new graph h. Determine the equation of h in its simplest form. (2)

[10]

QUESTION 8

In the diagram, A, B and C lie in the same horizontal plane with AB = AC. D is directly above A such that 2AD = BC. Also, AD = p, $A\hat{B}C = x$ and $D\hat{B}A = y$.



8.1 Determine AB in terms of p and y. (2)

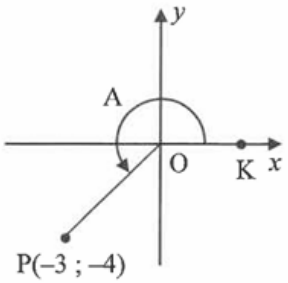
8.2 Show that $\cos x = \tan y$. (4)

8.3 If $x = 60°$, calculate the size of y. (2)
[8]

Paper F

QUESTION 5

5.1 In the diagram, line OP is given with P(–3 ; –4). $K\hat{O}P = A$.



Determine, **without using a calculator**, the value of:

5.1.1 $\cos A$ (2)

5.1.2 $\cos 2A$ (2)

5.1.3 $\sin(A-B)$, if it is further given that $\sin B = \frac{4}{5}$ and $90° < B < 360°$ (4)

5.2 If $\cos\alpha = p$, express the following expression in terms of p:

$$\frac{\cos\left(\frac{\alpha}{2} - 45°\right)\sin\left(\frac{\alpha}{2} - 45°\right)}{2}$$

(4)
[12]

QUESTION 6

6.1 Given the identity: $\cos(x-y)=\cos x\cos y+\sin x\sin y$

6.1.1 Use the compound angle identity given above to derive a formula for $\cos(x+y)$. (2)

6.1.2 Hence, or otherwise, show that:

$$\frac{\cos(90°-x)\cos y+\sin(-y)\cos(180°+x)}{\cos x\cos(360°+y)+\sin(360°-x)\sin y}=\tan(x+y)$$ (6)

6.2 Given: $f(x)=\sqrt{6\sin^2 x-11\cos(90°+x)+7}$

Solve for x in the interval $x\in(0°\,;\,360°)$ if $f(x)=2$. (6)

6.3 Consider the function: $g(x)=\dfrac{4-8\sin^2 x}{3}$

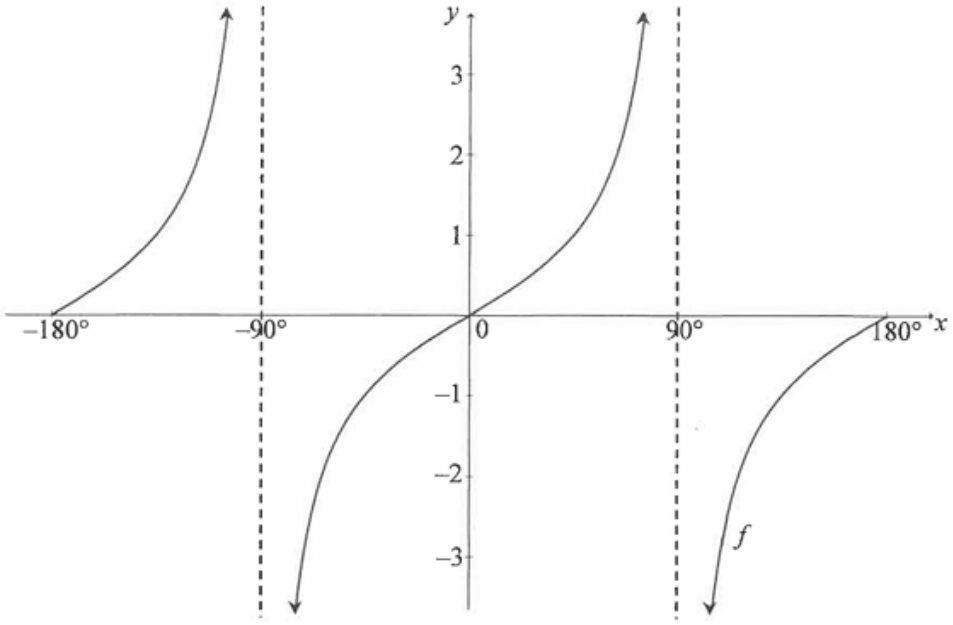
6.3.1 Calculate the maximum value of g. (3)

6.3.2 Write down the smallest possible value of x for which g will have a maximum value in the interval $x\in(0°\,;\,360°]$. (1)

[18]

QUESTION 7

In the diagram below, the graph of $f(x)=\tan x$ is drawn for the interval $x\in[-180°\,;\,180°]$.



7.1 Write down the equation of the asymptote of f in the interval $x\in[0°\,;\,180°]$. (1)

7.2 Write down the values of x in the interval $x\in[-180°\,;\,0°]$ for which $f(x)\le 0$. (2)

7.3 Given: $g(x)=\cos 2x+1$

7.3.1 Write down the period of g. (1)

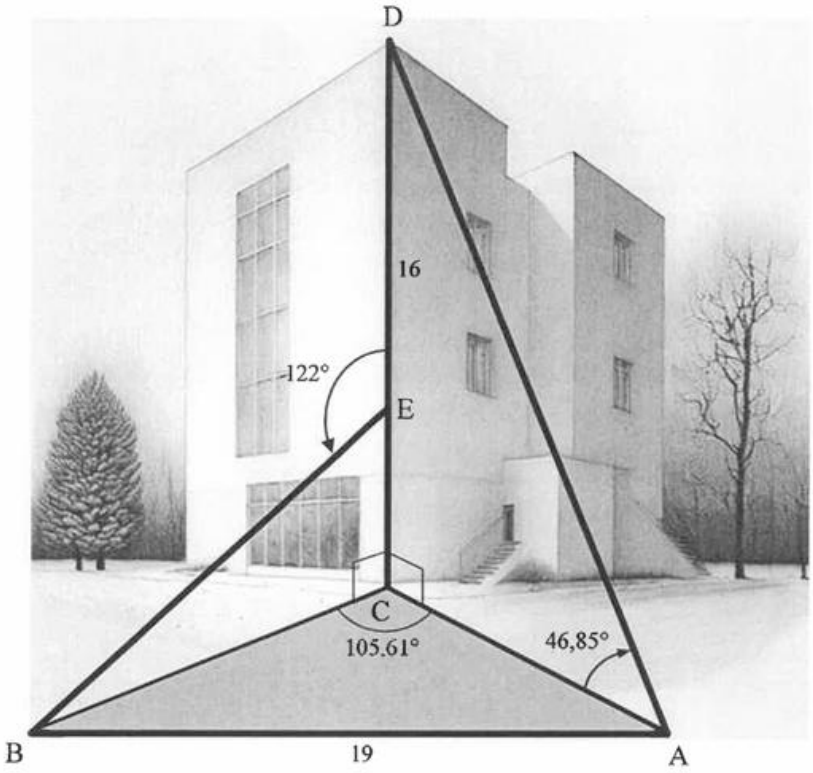
7.3.2 On the grid given in the ANSWER BOOK, draw the graph of $g(x)=\cos 2x+1$ for the interval $x\in[-180°\,;180°]$. Clearly show the intercepts with the axes as well as the coordinates of the turning points. (3)

7.4 Use the graphs to determine the general solution of $2\cos^3 x-\sin x=0$. (4)

[11]

QUESTION 8

In the diagram, C is the foot of a vertical building and D is the top of the same building. The height of the building, CD, is 16 m. Two observers are standing 19 m apart at points A and B, where A, B and C lie in the same horizontal plane. A painter is working at point E on the building. The angle of elevation of D from A is 46,85°. $D\hat{E}B = 122°$ and $B\hat{C}A = 105{,}61°$.



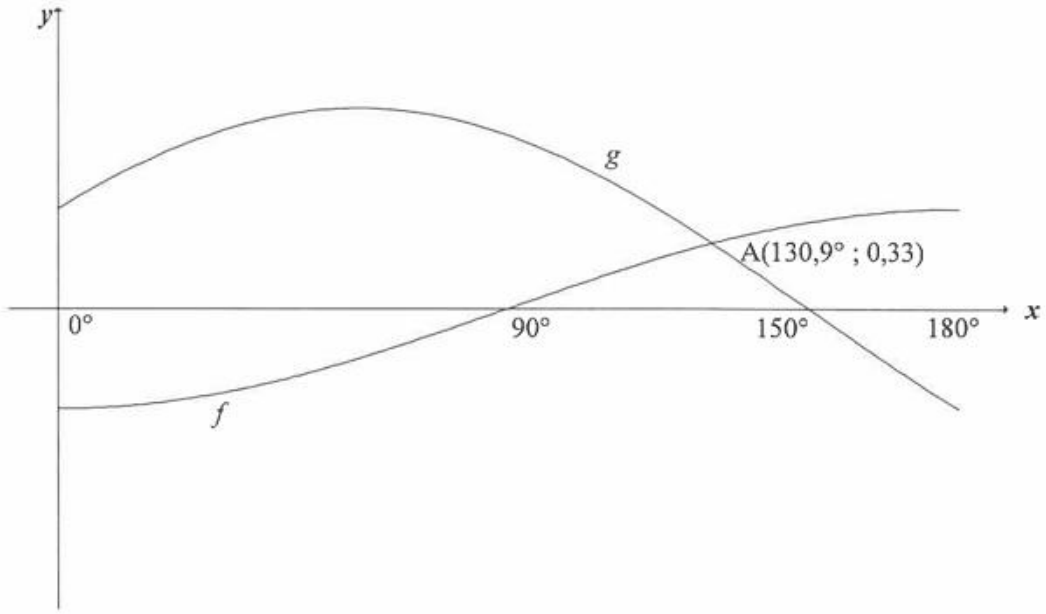
8.1 Calculate the length of AC, the distance between the observer at A and the foot of the building. (2)

8.2 Calculate how far the painter at E is from the top of the building. (7)
[9]

Paper G

QUESTION 5

The graphs of $f(x)=-\frac{1}{2}\cos x$ and $g(x)=\sin(x+30°)$, for the interval $x\in[0°;180°]$, are drawn below. A(130,9° ; 0,33) is the approximate point of intersection of the two graphs.



5.1 Write down the period of g. (1)

5.2 Write down the amplitude of f (1)

5.3 Determine the value of $f(180°)-g(180°)$ (1)

5.4 Use the graphs to determine the values of x, in the interval $x\in[0°;180°]$, for which:

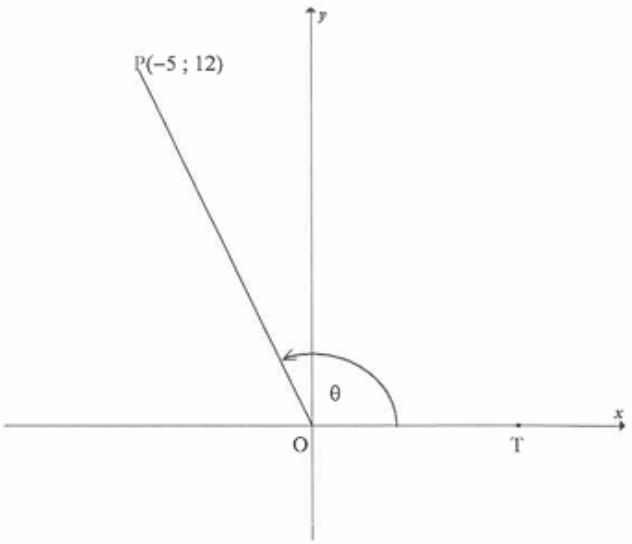
5.4.1 $f(x-10°)=g(x-10°)$ (1)

5.4.2 $\sqrt{3}\sin x+\cos x\geq 1$ (4)

[8]

QUESTION 6

6.1 In the diagram, P(–5 ; 12) and T lies on the positive x-axis. $P\hat{O}T = \theta$



Answer the following **without using a calculator**:

6.1.1 Write down the value of $\tan\theta$ (1)

6.1.2 Calculate the value of $\cos\theta$ (3)

6.1.3 S(a ; b) is a point in the third quadrant such that $T\hat{O}S = \theta + 90°$ and OS = 6,5 units. Calculate the value of b. (4)

6.2 Determine, **without using a calculator**, the value of the following trigonometric expression:

$$\frac{\sin 2x.\cos(-x) + \cos 2x.\sin(360° - x)}{\sin(180° + x)}$$ (5)

6.3 Determine the general solution of the following equation:

$$6\sin^2 x + 7\cos x - 3 = 0$$ (6)

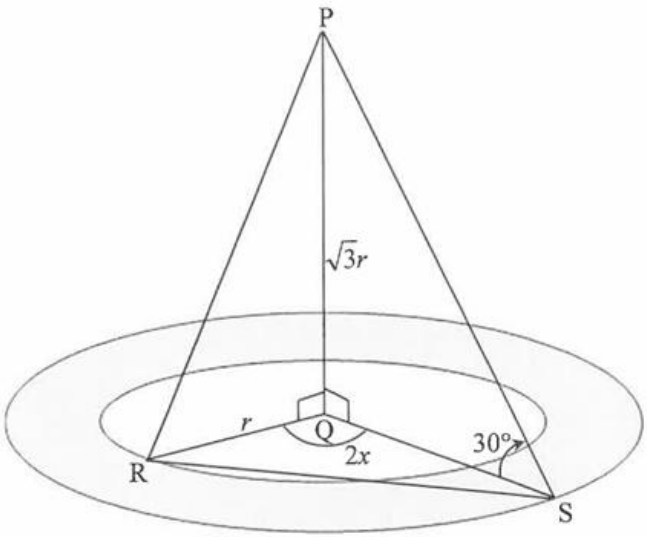
6.4 Given: $x + \frac{1}{x} = 3\cos A$ and $x^2 + \frac{1}{x^2} = 2$

Determine the value of $\cos 2A$ **without using a calculator**. (5)
[24]

QUESTION 7

A landscape artist plans to plant flowers within two concentric circles around a vertical light pole PQ. R is a point on the inner circle and S is a point on the outer circle. R, Q and S lie in the same horizontal plane. RS is a pipe used for the irrigation system in the garden.

- The radius of the inner circle is r units and the radius of the outer circle is QS.
- The angle of elevation from S to P is 30°.
- $R\hat{Q}S = 2x$ and $PQ = \sqrt{3}r$



7.1 Show that $QS = 3r$ (3)

7.2 Determine, in terms of r, the area of the flower garden. (2)

7.3 Show that $RS = r\sqrt{10 - 6\cos 2x}$ (3)

7.4 If $r = 10$ metres and $x = 56°$, calculate RS. (2)

[10]

Paper H

QUESTION 5

5.1 **Without using a calculator**, simplify the following expression to ONE trigonometric ratio:

$$\frac{\sin 140°.\sin(360° - x)}{\cos 50°.\tan(-x)} \quad (6)$$

5.2 Prove the identity: $\frac{-2\sin^2 x + \cos x + 1}{1 - \cos(540° - x)} = 2\cos x - 1$ (4)

5.3 Given: $\sin 36° = \sqrt{1 - p^2}$

Without using a calculator, determine EACH of the following in terms of p:

5.3.1 $\tan 36°$ (3)

5.3.2 $\cos 108°$ (4)

[17]

QUESTION 6

6.1 Given: $\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

6.1.1 Use the given identity to derive a formula for $\cos(\alpha + \beta)$ (3)

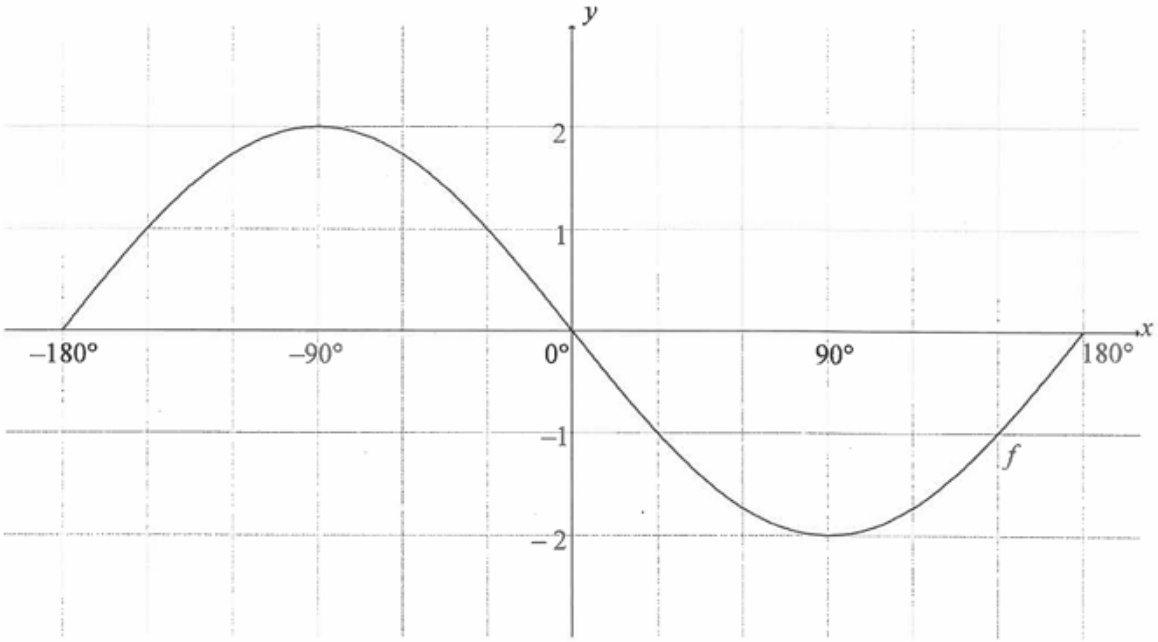
6.1.2 Simplify completely: $2\cos 6x\cos 4x - \cos 10x + 2\sin^2 x$ (5)

6.2 Determine the general solution of $\tan x = 2\sin 2x$ where $\cos x < 0$. (7)

[15]

QUESTION 7

In the diagram below, the graph of $f(x) = -2\sin x$ is drawn for the interval $x \in [-180° ; 180°]$.



7.1 On the grid provided in the ANSWER BOOK, draw the graph of $g(x) = \cos(x - 60°)$ for $x \in [-180° ; 180°]$. Clearly show ALL intercepts with the axes and turning points of the graph. (3)

7.2 Write down the period of $f(3x)$. (2)

7.3 **Use the graphs** to determine the value of x in the interval $x \in [-180° ; 180°]$ for which $f(x) - g(x) = 1$. (1)

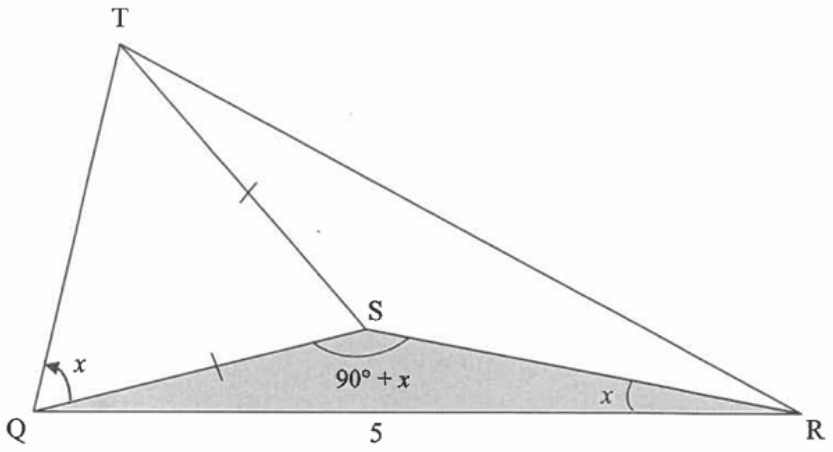
7.4 Write down the range of k, if $k(x) = \frac{1}{2}g(x) + 1$. (2)

[8]

QUESTION 8

In the diagram below, T is a hook on the ceiling of an art gallery. Points Q, S and R are on the same horizontal plane from where three people are observing the hook T. The angle of elevation from Q to T is x.

$Q\hat{S}R = 90° + x$, $Q\hat{R}S = x$, QR = 5 units and TS = SQ.



8.1 Prove that $QS = 5 \tan x$ (3)

8.2 Prove that the length of $QT = 10 \sin x$ (5)

8.3 Calculate the area of ΔTQR if $T\hat{Q}R = 70°$ and $x = 25°$. (2)

[10]

Paper I

QUESTION 5

5.1 Given: $\sin\beta = \frac{1}{3}$, where $\beta \in (90° ; 270°)$

Without using a calculator, determine each of the following:

5.1.1 $\cos\beta$ (3)

5.1.2 $\sin 2\beta$ (3)

5.1.3 $\cos(450° - \beta)$ (3)

5.2 Given: $\frac{\cos^4 x + \sin^2 x.\cos^2 x}{1+\sin x}$

5.2.1 Prove that $\frac{\cos^4 x + \sin^2 x.\cos^2 x}{1+\sin x} = 1 - \sin x$ (4)

5.2.2 For what value(s) of x in the interval $x \in [0° ; 360°]$ is $\frac{\cos^4 x + \sin^2 x.\cos^2 x}{1+\sin x}$ undefined? (2)

5.2.3 Write down the minimum value of the function defined by $y = \frac{\cos^4 x + \sin^2 x.\cos^2 x}{1+\sin x}$ (2)

5.3 Given: $\cos(A - B) = \cos A\cos B + \sin A\sin B$

5.3.1 Use the above identity to deduce that $\sin(A - B) = \sin A\cos B - \cos A\sin B$ (3)

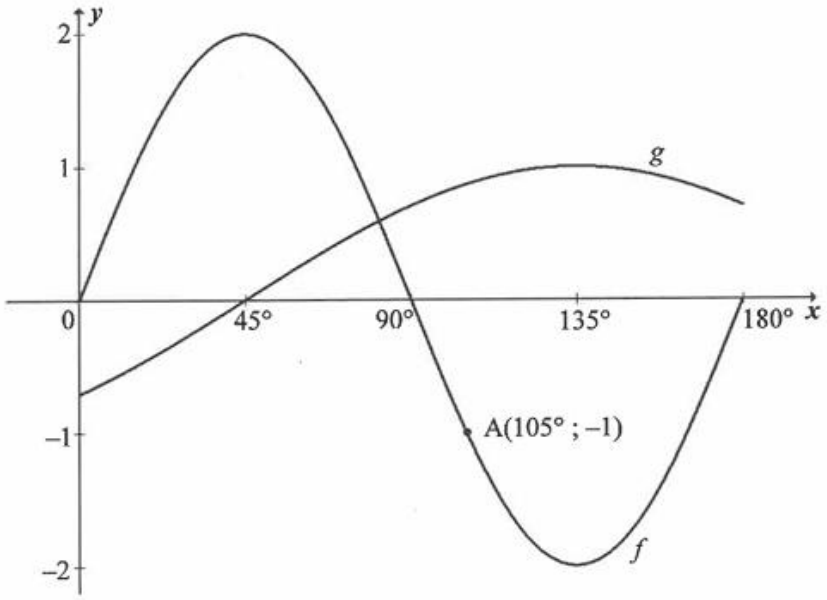
5.3.2 Hence, or otherwise, determine the general solution of the equation $\sin 48°\cos x - \cos 48°\sin x = \cos 2x$ (5)

5.4 Simplify $\frac{\sin 3x + \sin x}{\cos 2x + 1}$ to a single trigonometric ratio. (6)

[31]

QUESTION 6

In the diagram, the graphs of $f(x)=2\sin 2x$ and $g(x)=-\cos(x+45°)$ are drawn for the interval $x\in[0°;180°]$. A(105° ; –1) lies on f.



6.1 Write down the period of f. (1)

6.2 Determine the range of g in the interval $x \in [0° ; 180°]$. (2)

6.3 Determine the values of x, in the interval $x \in [0° ; 180°]$, for which:

6.3.1 $f(x).g(x)>0$ (2)

6.3.2 $f(x)+1\leq 0$ (2)

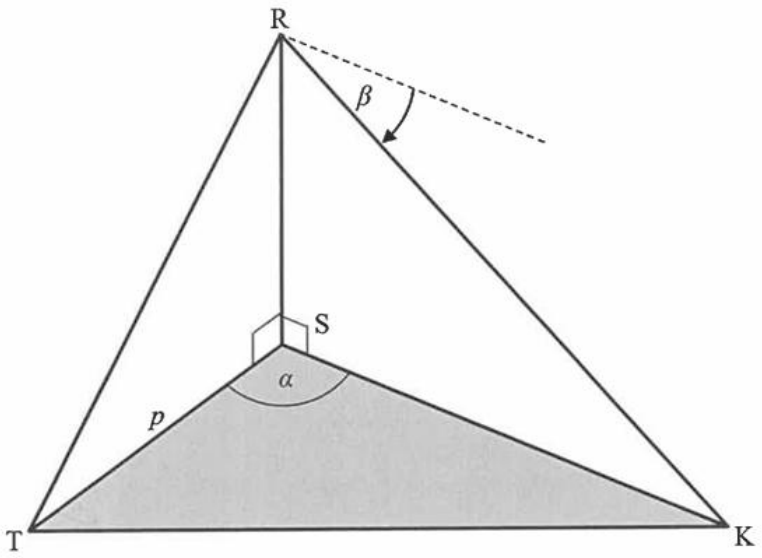
6.4 Another graph p is defined as $p(x) = -f(x)$. D(k ; –1) lies on p. Determine the value(s) of k in the interval $x \in [0° ; 180°]$. (3)

6.5 Graph h is obtained when g is translated 45° to the left. Determine the equation of h. Write your answer in its simplest form. (2)

[12]

QUESTION 7

In the diagram, S, T and K lie in the same horizontal plane. RS is a vertical tower. The angle of depression from R to K is β. $\hat{TSK} = \alpha$, TS = p metres and the area of ΔSTK is q m^2.



7.1 Determine the length of SK in terms of p, q and α. (2)

7.2 Show that $RS = \dfrac{2q \tan \beta}{p \sin \alpha}$ (2)

7.3 Calculate the size of α if $\alpha < 90°$ and RS = 70 m, p = 80 m, q = 2 500 m^2 and $\beta = 42°$. (3)
[7]

Paper J

QUESTION 5

5.1 If $\sin 40° = p$, write EACH of the following in terms of p.

5.1.1 $\sin 220°$ (2)

5.1.2 $\cos^2 50°$ (2)

5.1.3 $\cos(-80°)$ (3)

5.2 Given: $\tan x(1-\cos^2 x)+\cos^2 x = \dfrac{(\sin x+\cos x)(1-\sin x\cos x)}{\cos x}$

5.2.1 Prove the above identity. (5)

5.2.2 For which values of x, in the interval $x \in [-180° ; 180°]$, will the identity be undefined? (3)

5.3 Given the expression: $\dfrac{\sin 150° + \cos^2 x - 1}{2}$

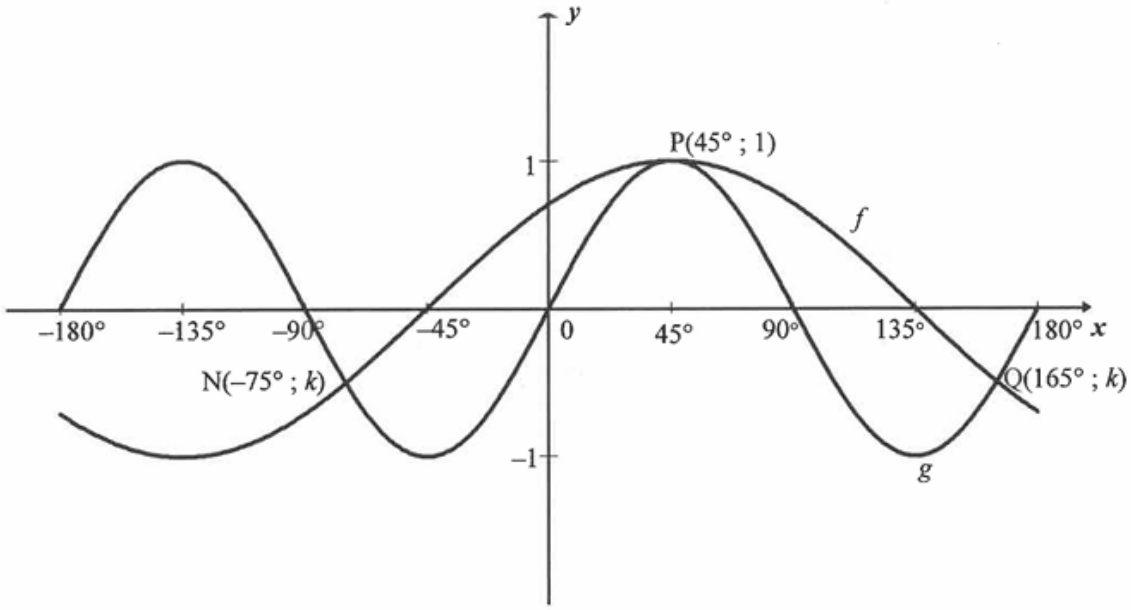
5.3.1 **Without using a calculator**, simplify the expression given above to a single trigonometric term in terms of $\cos 2x$. (6)

5.3.2 Hence, determine the general solution of $\dfrac{\sin 150° + \cos^2 x - 1}{2} = \dfrac{1}{25}$ (5)

[26]

QUESTION 6

In the diagram, the graphs of $f(x)=\cos(x+a)$ and $g(x)=\sin 2x$ are drawn for the interval $x\in[-180°;180°]$. The graphs intersect at N(–75° ; k), P(45° ; 1) and Q(165° ; k). P is also a turning point of both graphs.



6.1 Write down the period of f. (1)

6.2 Write down the amplitude of g. (1)

6.3 Write down the value of a. (1)

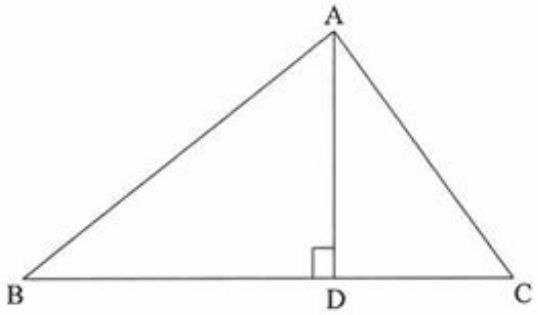
6.4 Calculate the value of k, the y-coordinate of N and Q, **without the use of a calculator**. (2)

6.5 Calculate the value of x if $g(x+60°)=f(x+60°)$ and $x\in[-45°;0°]$. (1)

6.6 **Without using a calculator**, determine the number of solutions the equation $\sqrt{2}\sin 2x=\sin x+\cos x$ has in the interval $x\in[-90°;90°]$. Clearly show ALL working. (4)
[10]

QUESTION 7

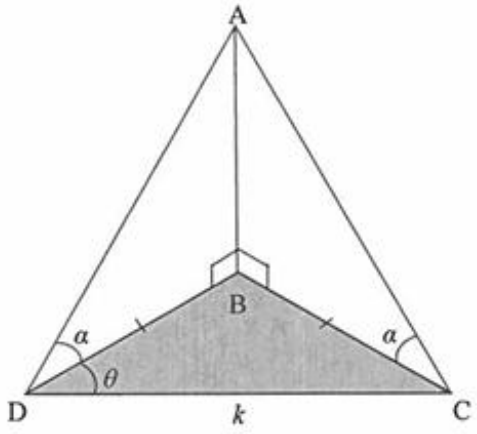
7.1 In the diagram, ΔABC is drawn. AD is drawn such that $AD \perp BC$.



7.1.1 Use the diagram above to determine AD in terms of $\sin\hat{B}$ (2)

7.1.2 Hence, prove that the area of $\Delta ABC = \frac{1}{2}(BC)(AB)\sin\hat{B}$ (1)

7.2 In the diagram, points B, C and D lie in the same horizontal plane. $A\hat{D}B = A\hat{C}B = \alpha$, $C\hat{D}B = \theta$ and $DC = k$ units. BD = BC.



7.2.1 Prove that AD = AC (2)

7.2.2 Prove that $BD = \frac{k}{2\cos\theta}$ (3)

7.2.3 Determine the area of ΔBCD in terms of k and a single trigonometric ratio of θ (3)

[11]

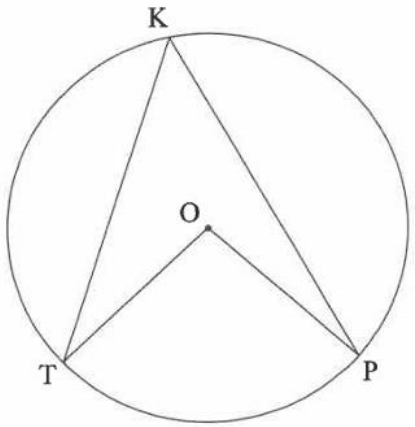
EUCLIDEAN GEOMETRY

Provide reason for ALL your statements in this section.

Paper A

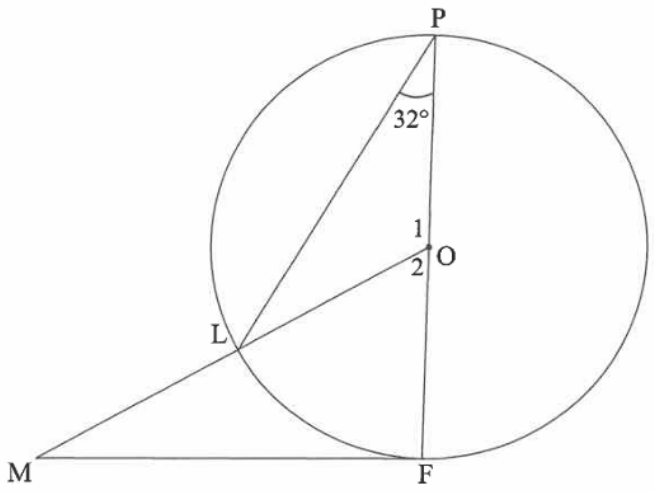
QUESTION 9

9.1 In the diagram, O is the centre of the circle. K, T and P lie on the circle.



Use the diagram above to prove the theorem which states that the angle subtended by a chord (or arc) at the centre of the circle is equal to twice the angle subtended by the same chord (or arc) at the circumference, that is prove that $T\hat{O}P = 2\,T\hat{K}P$. (5)

9.2 In the diagram, O is the centre of the circle. POF is the diameter of the circle and MF is a tangent to the circle at F. OM cuts the circle at L. $\hat{P} = 32°$.



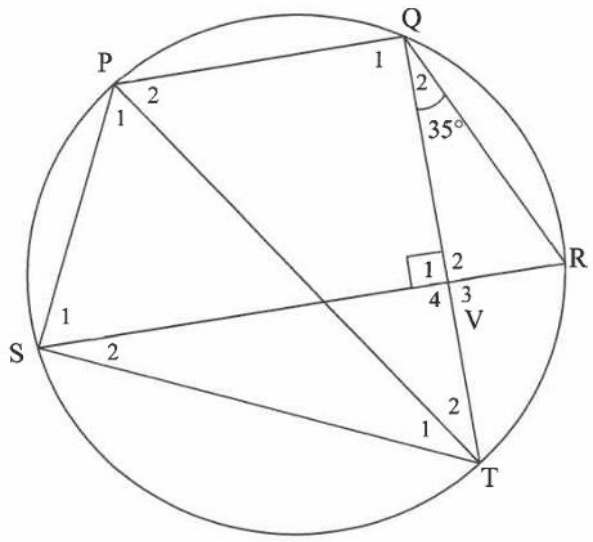
Calculate, with reasons, the size of:

9.2.1 $\hat{Q}_2$ (2)

9.2.2 $\hat{M}$ (3)
[10]

QUESTION 10

In the diagram, PQRS is a cyclic quadrilateral. T is a point on the circle such that QT is perpendicular to SR at V. PT and ST are drawn. $\hat{Q}_2 = 35°$ and $\hat{R} = \hat{S}_1$.



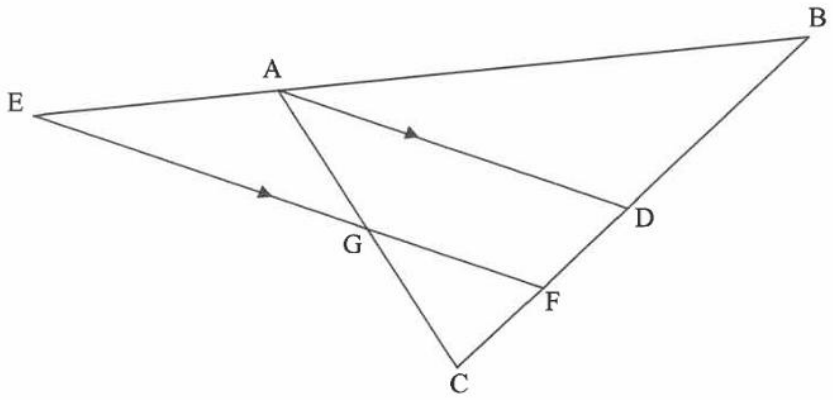
10.1 Calculate, with reasons, the size of $Q\hat{T}S$. (3)

10.2 Prove that PQ || SR. (3)

10.3 Prove that PT is a diameter of the circle. (2)
[8]

QUESTION 11

11.1 In the diagram, ΔABC is drawn. BA is produced to E. F and D are points on BC such that $AD \parallel EF$. AC and EF intersect at G. $\frac{CF}{FB}=\frac{2}{5}$ and $\frac{CG}{GA}=\frac{3}{2}$.



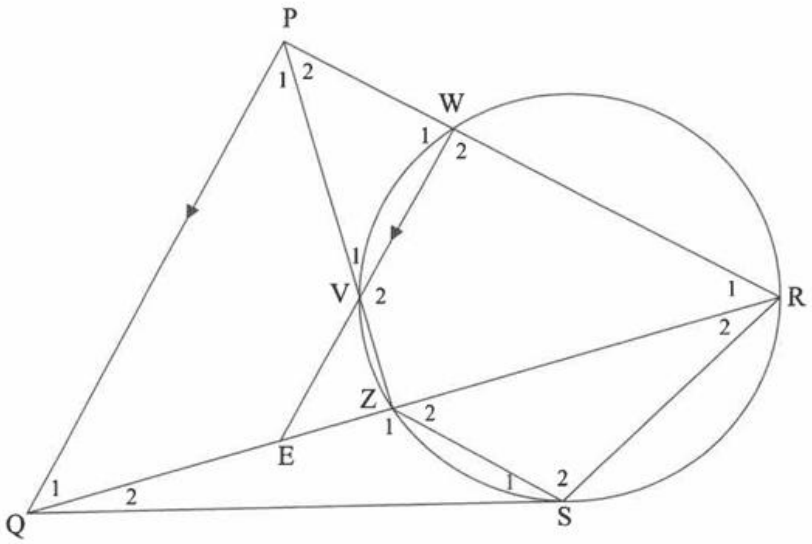
Calculate, with reasons, the value of:

11.1.1 $\frac{FD}{CF}$ (2)

11.1.2 $\frac{BA}{EA}$ (4)

11.1.3 $\frac{\text{Area of } \Delta GCF}{\text{Area of GFDA}}$ (4)

11.2 In the diagram, WVZR is a cyclic quadrilateral. RZ is produced to Q. A tangent is drawn from Q to touch the circle at S. WV is produced to E, a point on ZQ. RW produced meets ZV produced in P. PQ || WE. RS and ZS are drawn.



Prove, with reasons, that:

11.2.1 $PR = \frac{PW.QR}{QE}$ (2)

11.2.2 If $\Delta PQZ \,|||\, \Delta RQP$, then $PQ^2 = RQ.QZ$ (1)

11.2.3 $\Delta QSZ \,|||\, \Delta QRS$ (3)

11.2.4 $PQ = QS$ (3)

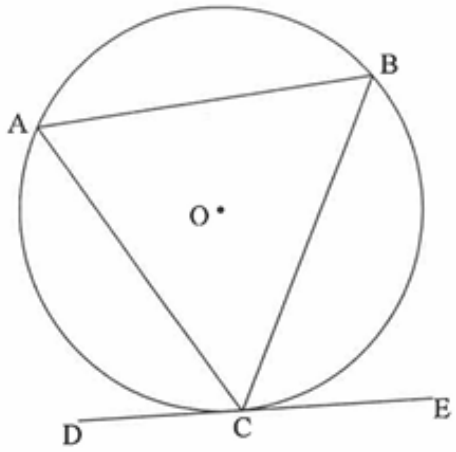
11.2.5 $PW = \frac{QE.PZ}{\sqrt{QR.QZ}}$ (4)

[23]

Paper B

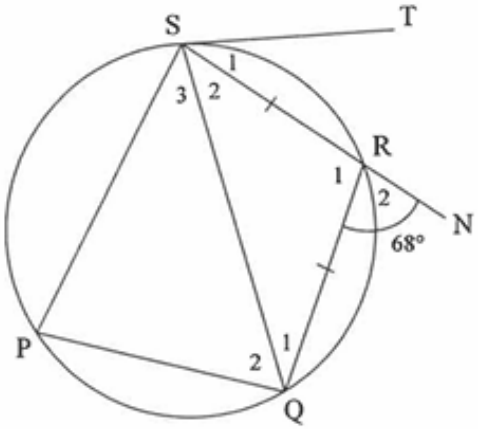
QUESTION 8

8.1 In the diagram, chords AB, BC and AC are drawn in the circle with centre O. DCE is a tangent to the circle at C.



Prove the theorem which states that the angle between the tangent to a circle and the chord drawn from the point of contact is equal to the angle in the alternate segment, i.e. $B\hat{C}E = \hat{A}$. (5)

8.2 In the diagram, PQRS is a cyclic quadrilateral with RQ = RS. ST is a tangent to the circle at S. SR is produced to N. $\hat{R}_2 = 68°$.



Determine, with reasons, the size of:

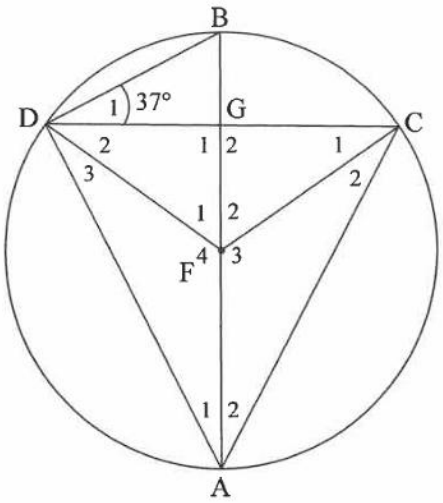
8.2.1 $\hat{P}$ (2)

8.2.2 $\hat{Q}_1$ (2)

8.2.3 $\hat{S}_1$ (2)
[11]

QUESTION 9

In the diagram, AB is a diameter of the circle, with centre F. AB and CD intersect at G. FD and FC are drawn. BA bisects $C\hat{A}D$ and $\hat{D}_1 = 37°$.



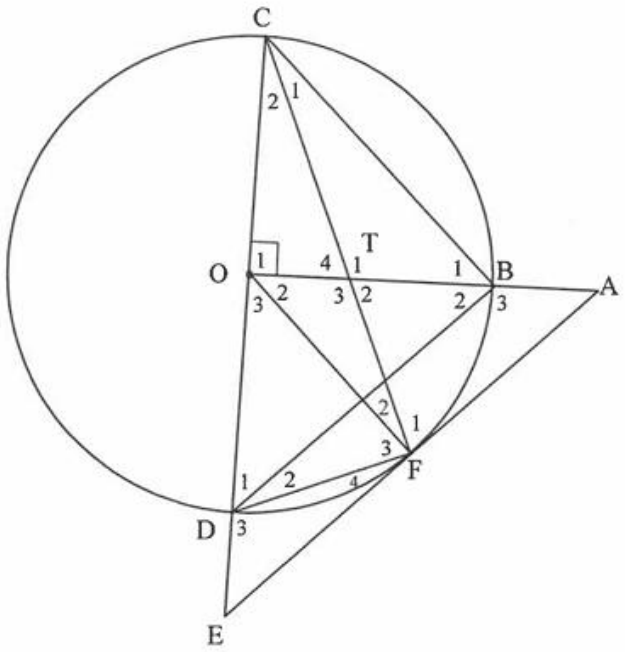
9.1 Determine, giving reasons, any three other angles equal to $\hat{D}_1$. (4)

9.2 Show that DG = GC. (4)

9.3 If it is further given that the radius of the circle is 20 units, calculate the length of BG. (4)
[12]

QUESTION 10

In the diagram, COD is the diameter of the circle with centre O. EA is a tangent to the circle at F. AO ⊥ CE. Diameter COD produced intersects the tangent to the circle at E. OB produced intersects the tangent to the circle at A. CF intersects OB in T. CB, BD, OF and FD are drawn.



Prove, with reasons, that:

10.1 TODF is a cyclic quadrilateral (4)

10.2 $\hat{D}_3 = \hat{T}_1$ (3)

10.3 $\Delta TFO \,|||\, \Delta DFE$ (5)

10.4 If $\hat{B}_2 = \hat{E}$, prove that DB || EA. (2)

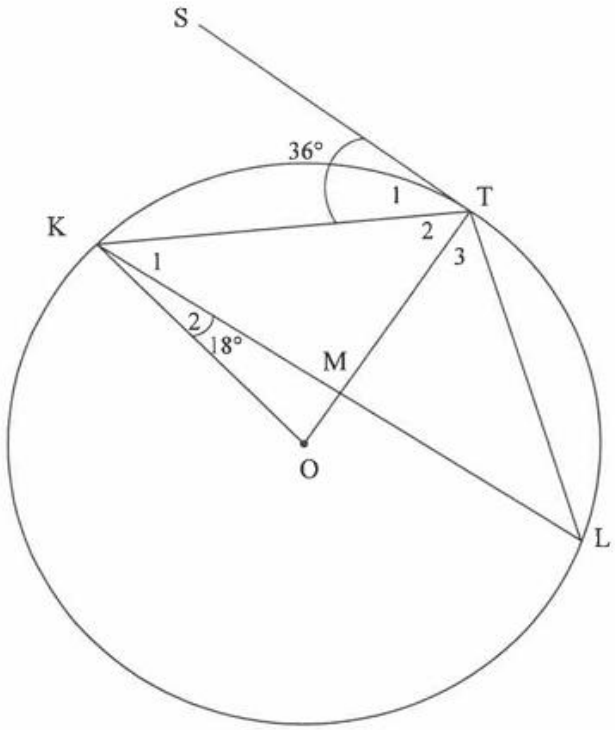
10.5 Prove that $DO = \dfrac{TO.FE}{AB}$ (5)

[19]

Paper C

QUESTION 8

8.1 In the diagram, O is the centre of the circle. K, T and L are points on the circle. KT, TL, KL, OK and OT are drawn. OT intersects KL at M. ST is a tangent to the circle at T. $S\hat{T}K = 36°$ and $O\hat{K}L = 18°$.



8.1.1 Determine, giving reasons, the size of:

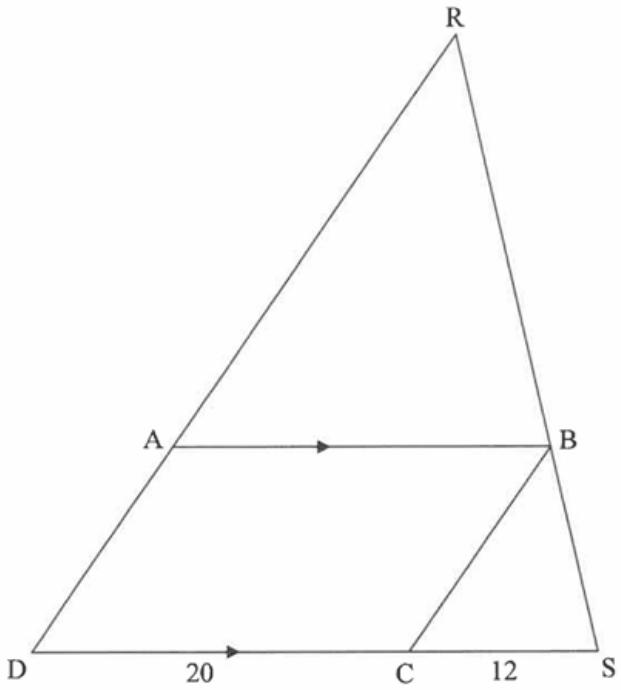
(a) $\hat{T}_2$ (2)

(b) $\hat{L}$ (2)

(c) $K\hat{O}T$ (2)

8.1.2 Prove, giving reasons, that KM = ML. (3)

8.2 In the diagram, ΔRDS is drawn. A, B and C are points on RD, RS and DS respectively such that AB || DS and RB : BS = 5 : 3. DC = 20 units and CS = 12 units.



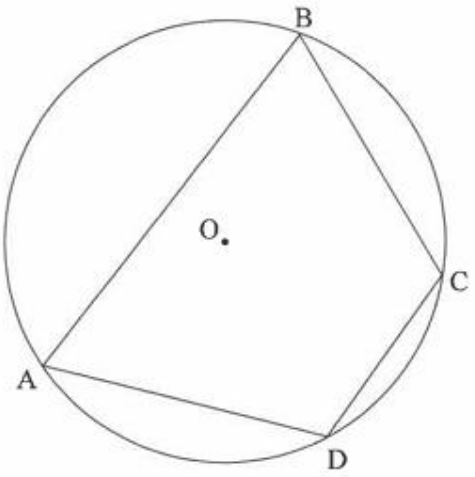
8.2.1 Prove, giving reasons, that BC || AD. (3)

8.2.2 If it is further given that RD = 48 units, calculate, giving reasons, the value of the ratio AD : AB. (3)

[15]

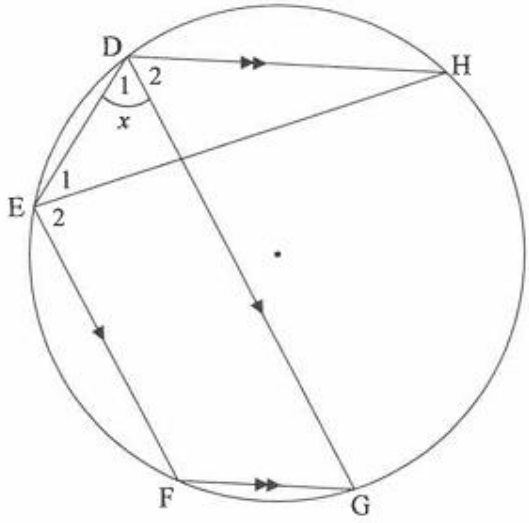
QUESTION 9

9.1 In the diagram, O is the centre of the circle. ABCD is a cyclic quadrilateral.



Use the diagram in the ANSWER BOOK to prove the theorem which states that the opposite angles of a cyclic quadrilateral are supplementary, that is prove that $\hat{B} + \hat{D} = 180°$. (5)

9.2 In the diagram, DEFG is a cyclic quadrilateral such that EF || DG. H is another point on the circle such that DH || FG. Chord EH is drawn. Let $\hat{D}_1 = x$.

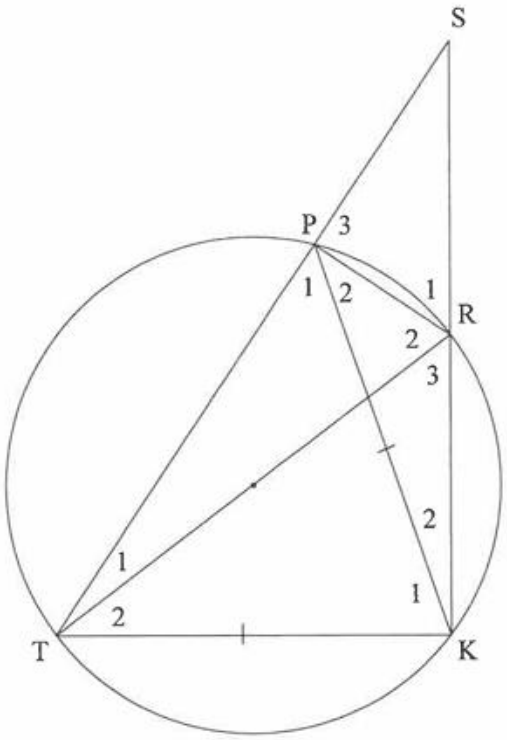


Prove, giving reasons, that $\hat{D}_1 = \hat{D}_2$. (4)

[9]

QUESTION 10

In the diagram, TR is a diameter of the circle. PRKT is a cyclic quadrilateral. Chords TP and KR are produced to intersect at S. Chord PK is drawn such that PK = TK.



10.1 Prove, giving reasons, that:

10.1.1 SR is a diameter of a circle passing through points S, P and R (4)

10.1.2 $\hat{S} = \hat{P}_2$ (5)

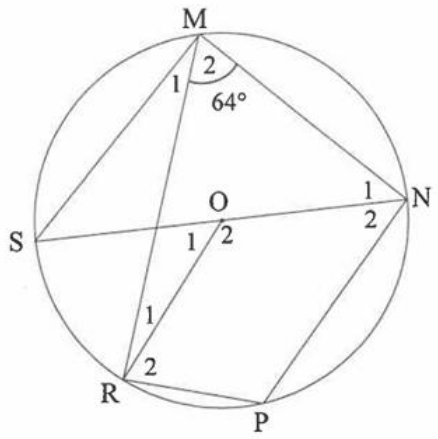
10.1.3 $\Delta SPK \,|||\, \Delta PRK$ (3)

10.2 If it is further given that SR = RK, prove that $ST = \sqrt{6}RK$. (5)
[17]

Paper D

QUESTION 8

8.1 In the diagram, O is the centre of the circle. MNPR is a cyclic quadrilateral and SN is a diameter of the circle. Chord MS and radius OR are drawn. $\hat{M}_2 = 64°$.



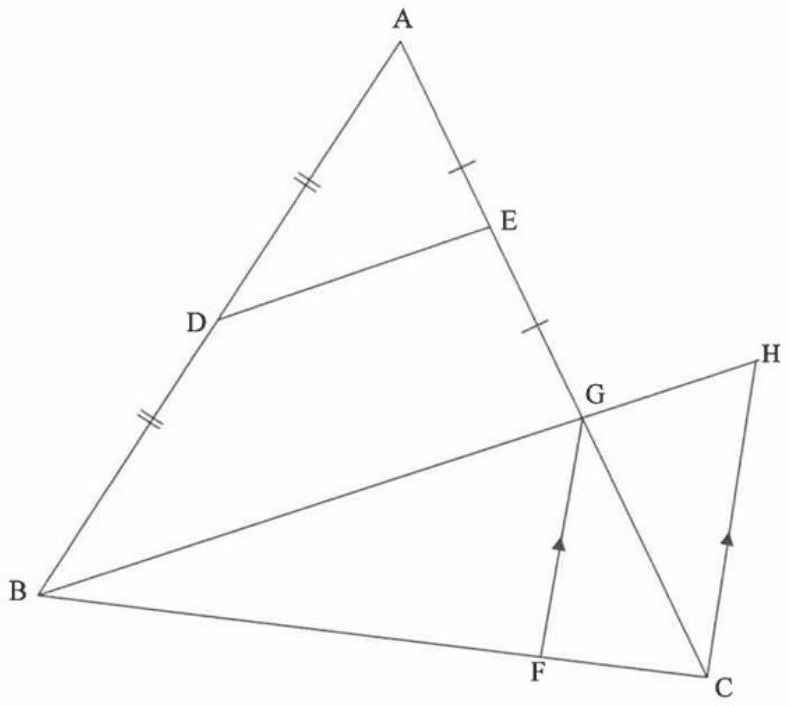
Determine, giving reasons, the size of the following angles:

8.1.1 $\hat{P}$ (2)

8.1.2 $\hat{M}_1$ (2)

8.1.3 $\hat{O}_1$ (2)

8.2 In the diagram, ΔABG is drawn. D and E are midpoints of AB and AG respectively. AG and BG are produced to C and H respectively. F is a point on BC such that FG || CH.



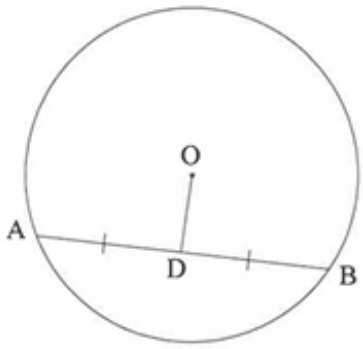
8.2.1 Give a reason why DE || BH. (1)

8.2.2 If it is further given that $\frac{FC}{BF}=\frac{1}{4}$, $DE = 3x - 1$ and $GH = x + 1$, calculate, giving reasons, the value of x. (6)

[13]

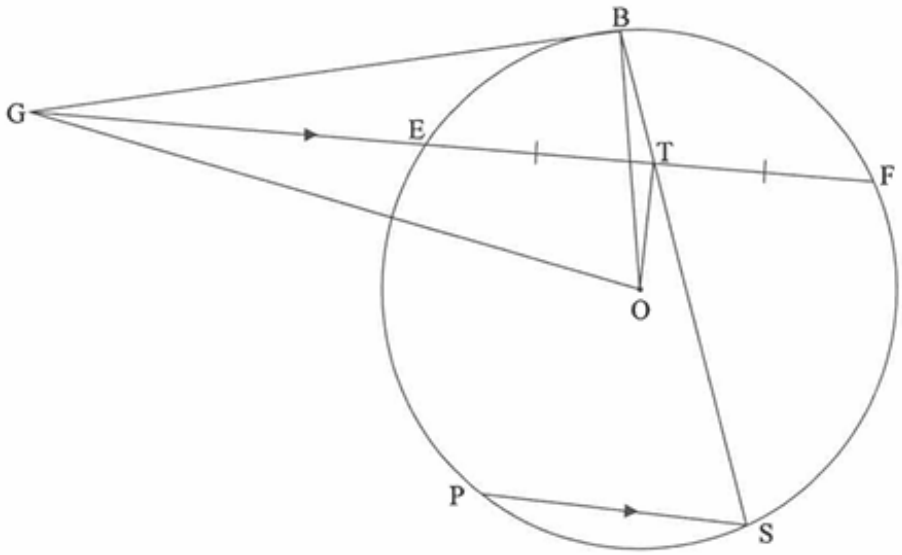
QUESTION 9

9.1 In the diagram, O is the centre of a circle. OD bisects chord AB.



Prove the theorem that states that the line from the centre of a circle that bisects a chord is perpendicular to the chord, i.e. OD ⊥ AB. (5)

9.2 In the diagram, E, B, F, S and P are points on the circle centred at O. GB is a tangent to the circle at B. FE is produced to meet the tangent at G. OT is drawn such that T is the midpoint of EF. GO and BO are drawn. BS is drawn through T. PS ∥ GF.



Prove, giving reasons, that:

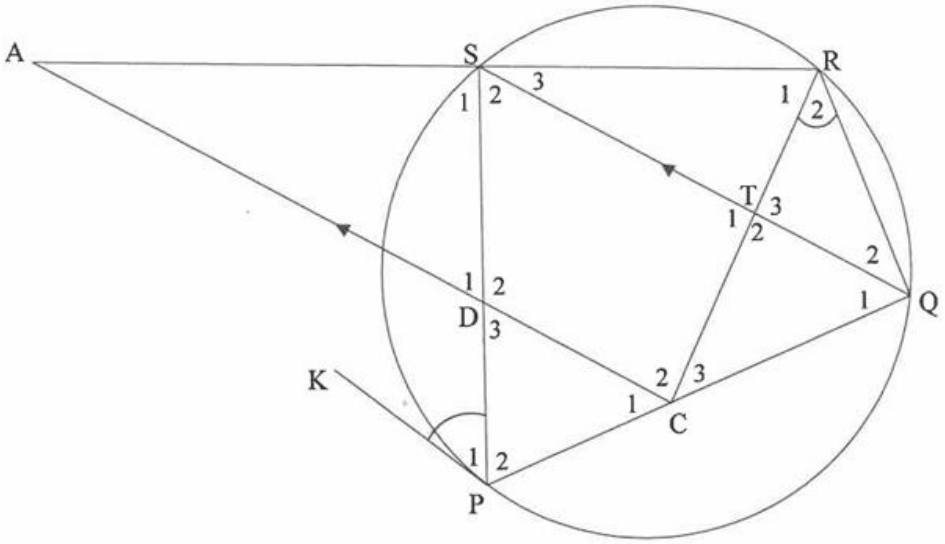
9.2.1 OTBG is a cyclic quadrilateral (5)

9.2.2 $G\hat{O}B = \hat{S}$ (4)

[14]

QUESTION 10

In the diagram, PQRS is a cyclic quadrilateral. KP is a tangent to the circle at P. C and D are points on chords PQ and PS respectively and CD produced meets RS produced at A. CA ∥ QS. RC is drawn. $\hat{P}_1 = \hat{R}_2$.



Prove, giving reasons, that:

10.1 $\hat{S}_1 = \hat{T}_2$ (4)

10.2 $\frac{AD}{AR} = \frac{AS}{AC}$ (5)

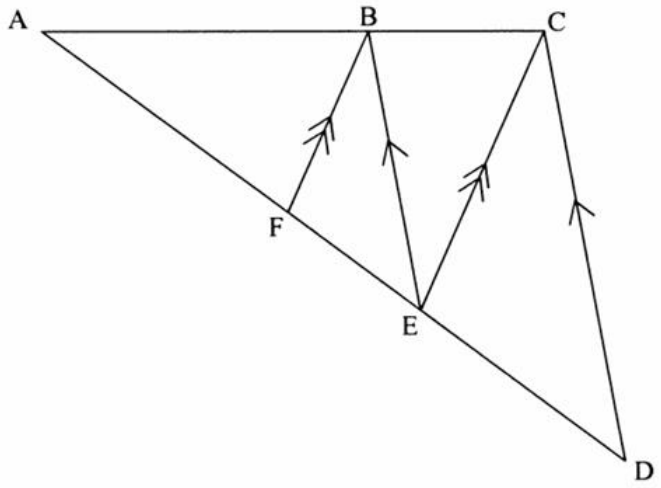
10.3 $AC \times SD = AR \times TC$ (4)

[13]

Paper E

QUESTION 9

In ΔADC, E is a point on AD and B is a point on AC such that EB || DC.
F is a point on AD such that FB || EC. It is also given that AB = 2BC.



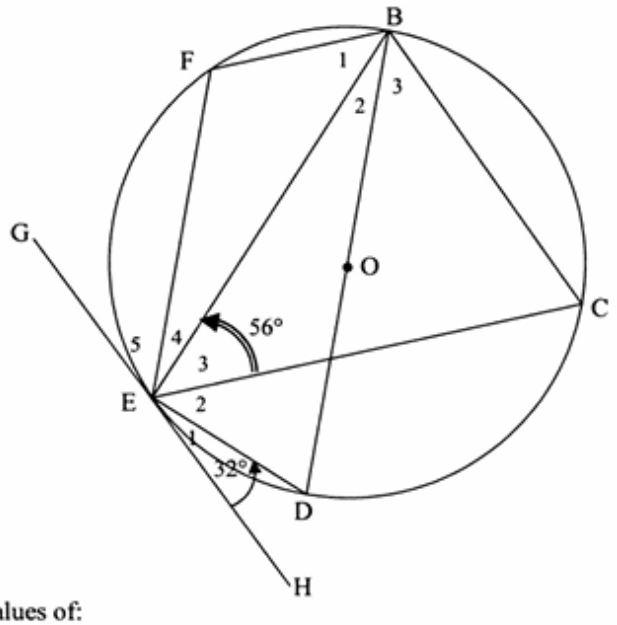
9.1 Determine the value of AF : FE (2)

9.2 Calculate the length of ED if AF = 8 cm. (4)
[6]

QUESTION 10

In the diagram below, O is the centre of the circle. BD is a diameter of the circle. GEH is a tangent to the circle at E. F and C are two points on the circle and FB, FE, BC, CE and BE are drawn.

$\hat{E}_1 = 32°$ and $\hat{E}_3 = 56°$.



Calculate, with reasons, the values of:

10.1 $\hat{E}_2$ (2)

10.2 $E\hat{B}C$ (3)

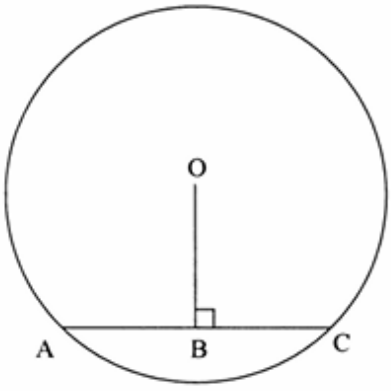
10.3 $\hat{F}$ (4)

[9]

QUESTION 11

In the diagram below, O is the centre of the circle and OB is perpendicular to the chord AC.

Prove, using Euclidean geometry methods, the theorem that states AB = BC.

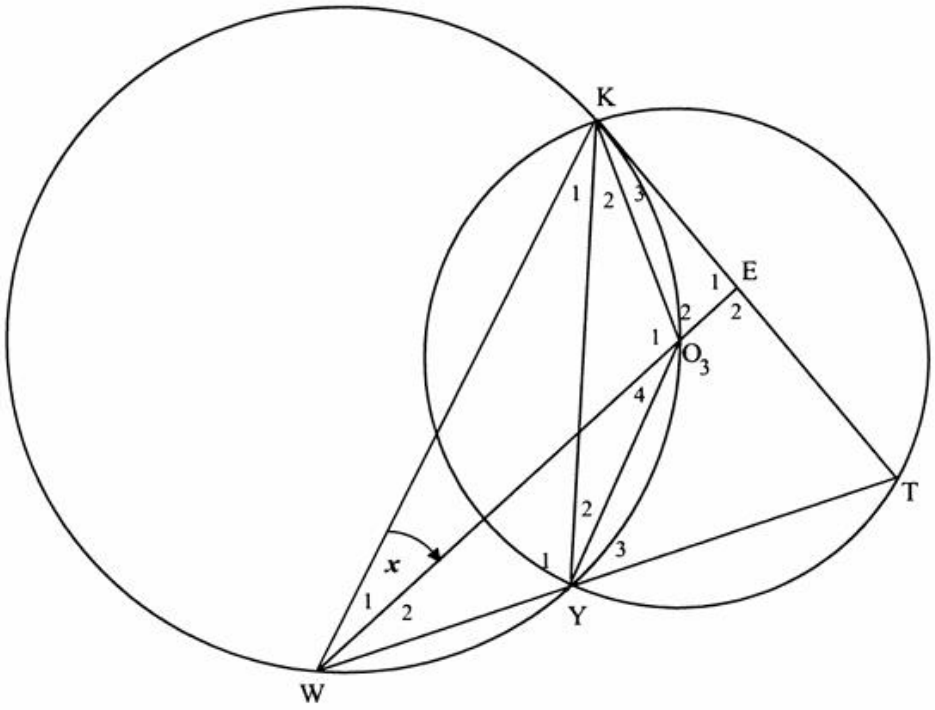


[5]

QUESTION 12

In the diagram below, two circles intersect at K and Y. The larger circle passes through O, the centre of the smaller circle. T is a point on the smaller circle such that KT is a tangent to the larger circle. TY produced meets the larger circle at W.
WO produced meets KT at E.

Let $\hat{W}_1 = x$



12.1 Determine FOUR other angles, each equal to x. (8)

12.2 Prove that $\hat{T} = 90° - x$. (3)

12.3 Prove that KE = ET. (3)

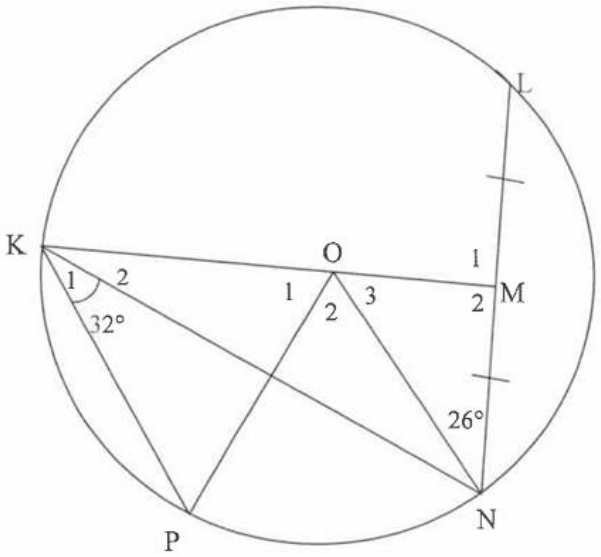
12.4 Prove that $KE^2 = OE.WE$ (6)

[20]

Paper F

QUESTION 8

8.1 O is the centre of the circle.. KOM bisects chord LN and $M\hat{N}O = 26°$. K and P are points on the circle with $N\hat{K}P = 32°$. OP is drawn.



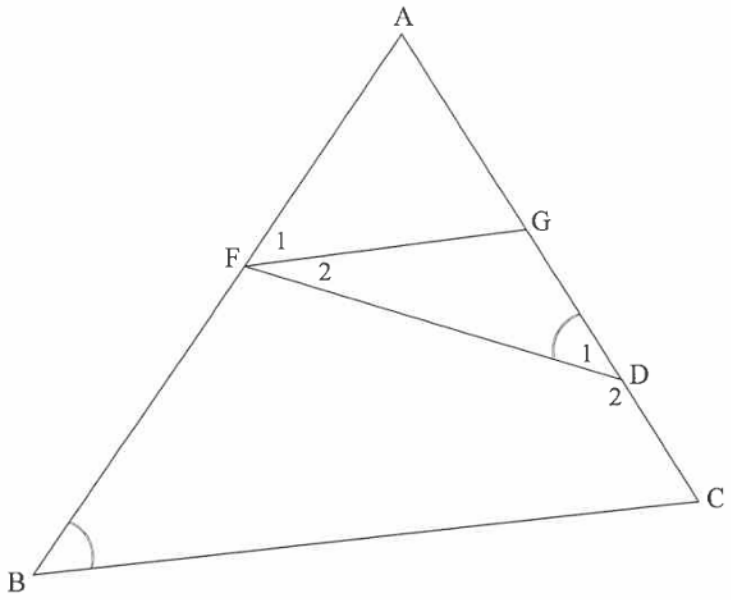
8.1.1 Determine, giving reasons, the size of:

(a) $\hat{O}_2$ (2)

(b) $\hat{O}_1$ (4)

8.1.2 Prove, giving reasons, that KN bisects $O\hat{K}P$. (3)

8.2 In ΔABC, F and G are points on sides AB and AC respectively. D is a point on GC such that $\hat{D}_1 = \hat{B}$.



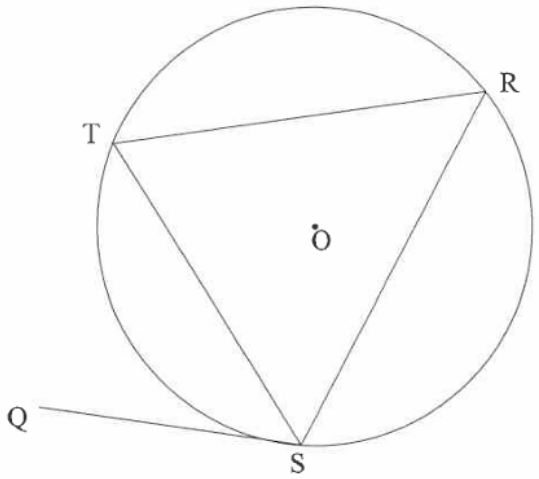
8.2.1 If AF is a tangent to the circle passing through points F, G and D, then prove, giving reasons, that FG || BC. (4)

8.2.2 If it is further given that $\frac{AF}{FB} = \frac{2}{5}$, $AC = 2x - 6$ and $GC = x + 9$, then calculate the value of x. (4)

[17]

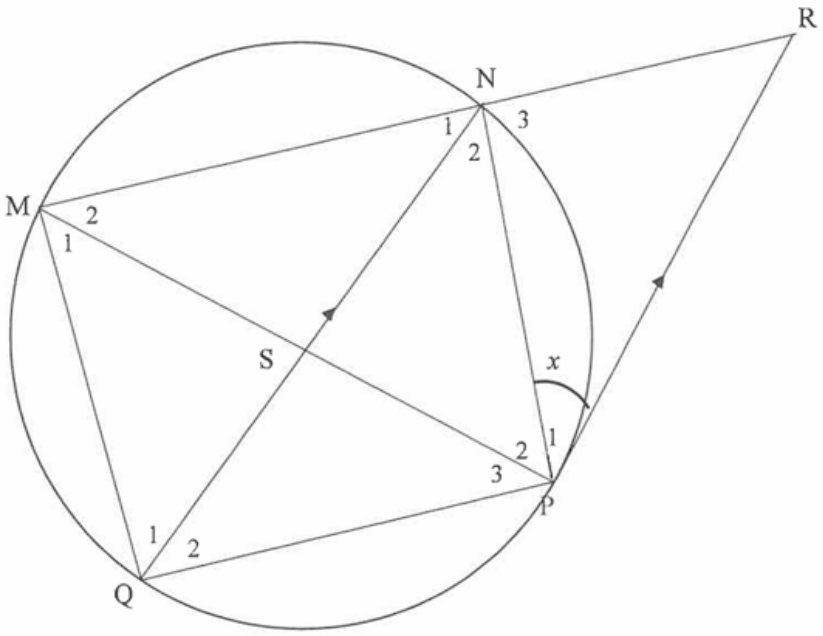
QUESTION 9

9.1 In the diagram, O is the centre of the circle. Points S, T and R lie on the circle. Chords ST, SR and TR are drawn in the circle. QS is a tangent to the circle at S.



Use the diagram to prove the theorem which states that $\hat{QST} = \hat{R}$. (5)

9.2 Chord QN bisects $M\hat{N}P$ and intersects chord MP at S. The tangent at P meets MN produced at R such that QN || PR. Let $\hat{P}_1 = x$.



9.2.1 Determine the following angles in terms of *x*. Give reasons

(a) $\hat{N}_2$ (2)

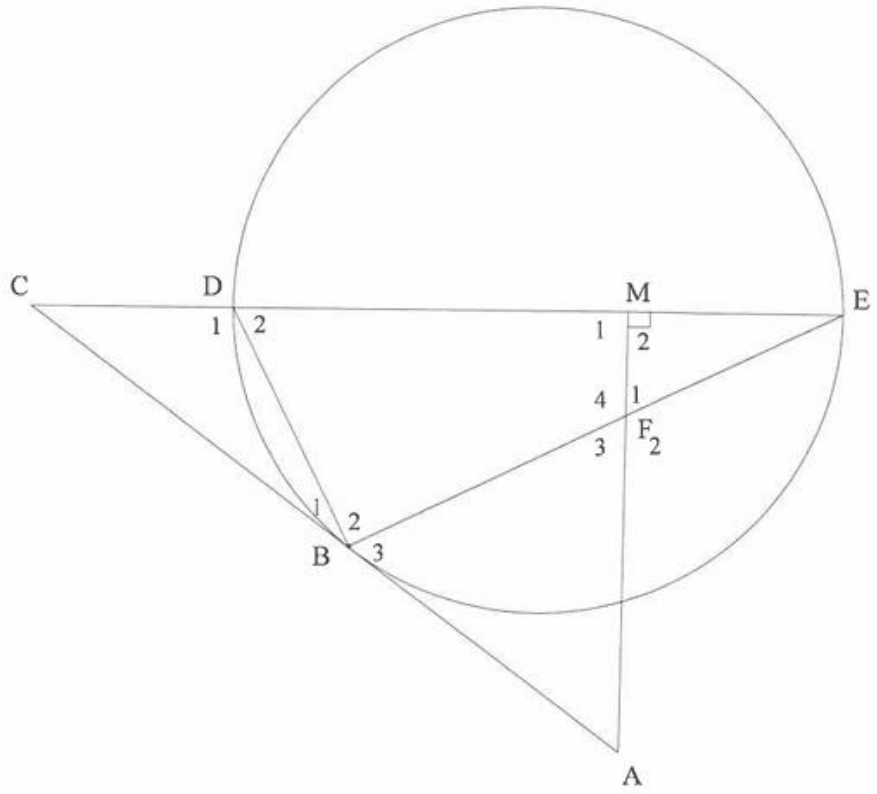
(b) $\hat{Q}_2$ (2)

9.2.2 Prove, giving reasons, that $\frac{MN}{NR} = \frac{MS}{SQ}$ (6)

[15]

QUESTION 10

In the diagram, a circle passes through D, B and E. Diameter ED of the circle is produced to C and AC is a tangent to the circle at B. M is a point on DE such that AM ⊥ DE. AM and chord BE intersect at F.



10.1 Prove, giving reasons, that:

10.1.1 FBDM is a cyclic quadrilateral (3)

10.1.2 $\hat{B}_3 = \hat{F}_1$ (4)

10.1.3 $\Delta CDB \;|||\; \Delta CBE$ (3)

10.2 If it is further given that CD = 2 units and DE = 6 units, calculate the length of:

10.2.1 BC (3)

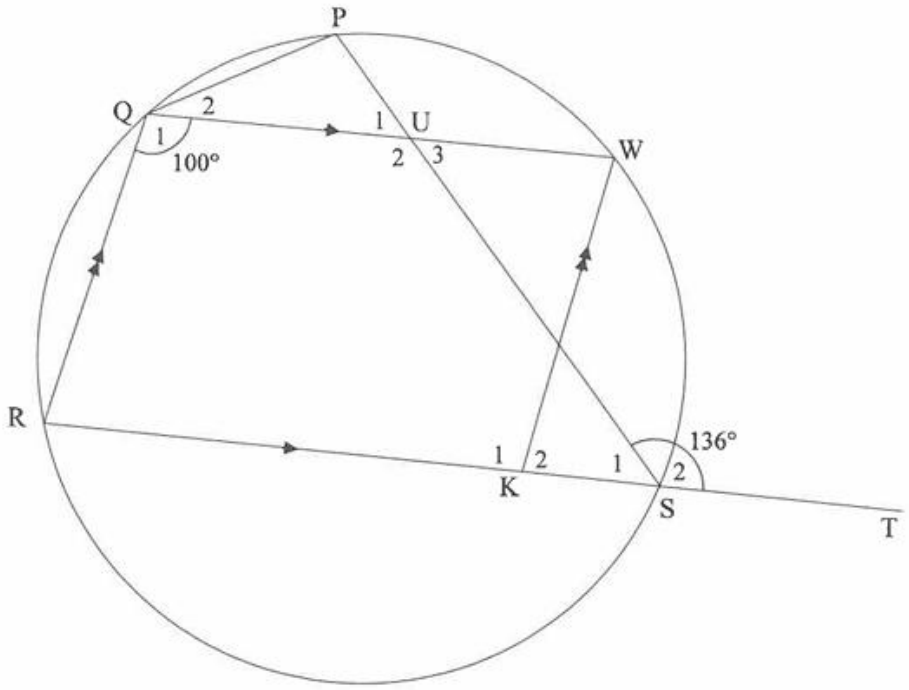
10.2.2 DB (4)

[17]

Paper G

QUESTION 8

8.1 In the diagram, PQRS is a cyclic quadrilateral. Chord RS is produced to T. K is a point on RS and W is a point on the circle such that QRKW is a parallelogram. PS and QW intersect at U. $P\hat{S}T = 136°$ and $\hat{Q}_1 = 100°$.



Determine, with reasons, the size of:

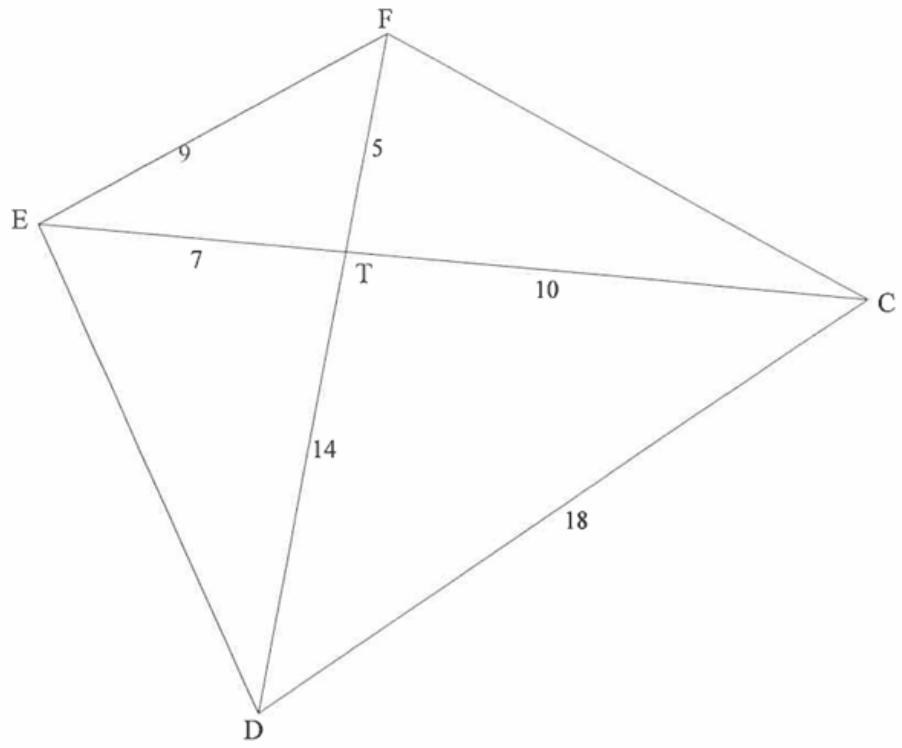
8.1.1 $\hat{R}$ (2)

8.1.2 $\hat{P}$ (2)

8.1.3 $P\hat{Q}W$ (3)

8.1.4 $\hat{U}_2$ (2)

8.2 In the diagram, the diagonals of quadrilateral CDEF intersect at T.
EF = 9 units, DC = 18 units, ET = 7 units, TC = 10 units, FT = 5 units and TD = 14 units.



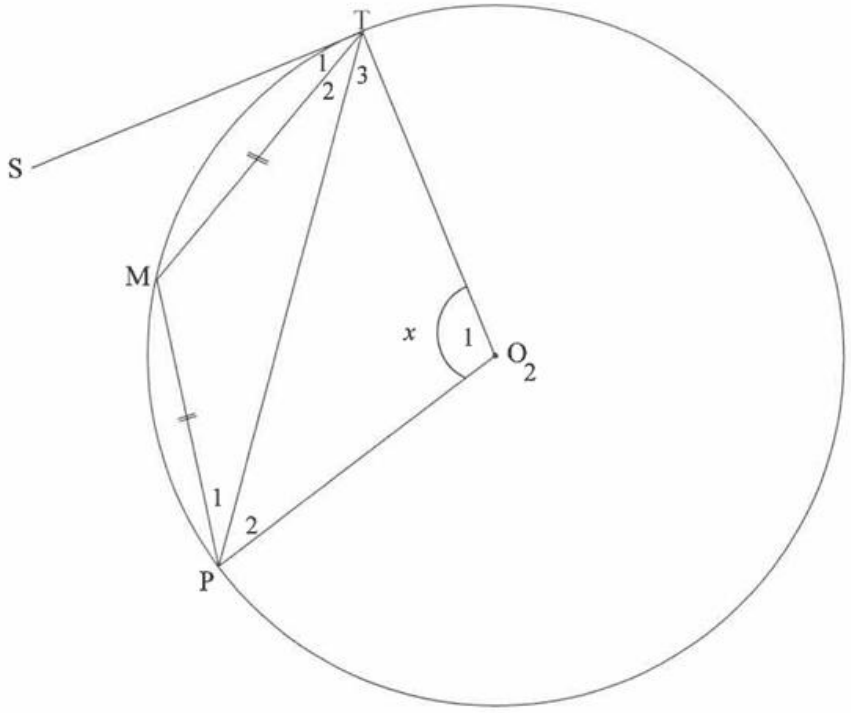
Prove, with reasons, that:

8.2.1 $E\hat{F}D = E\hat{C}D$ (4)

8.2.2 $D\hat{F}C = D\hat{E}C$ (3)
[16]

QUESTION 9

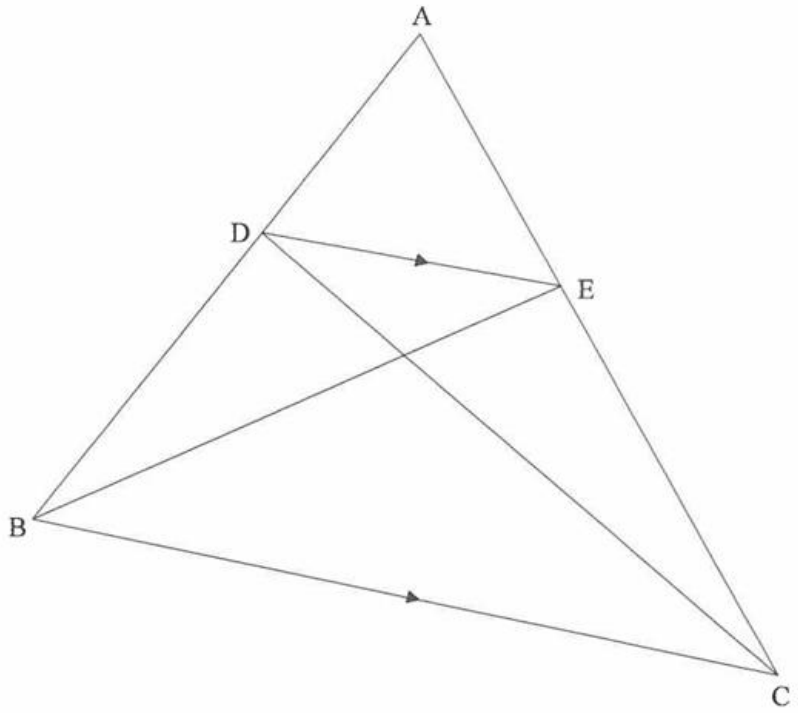
In the diagram, O is the centre of the circle. ST is a tangent to the circle at T. M and P are points on the circle such that TM = MP. OT, OP and TP are drawn. Let $\hat{O}_1 = x$.



Prove, with reasons, that $S\hat{T}M = \frac{1}{4}x$. [7]

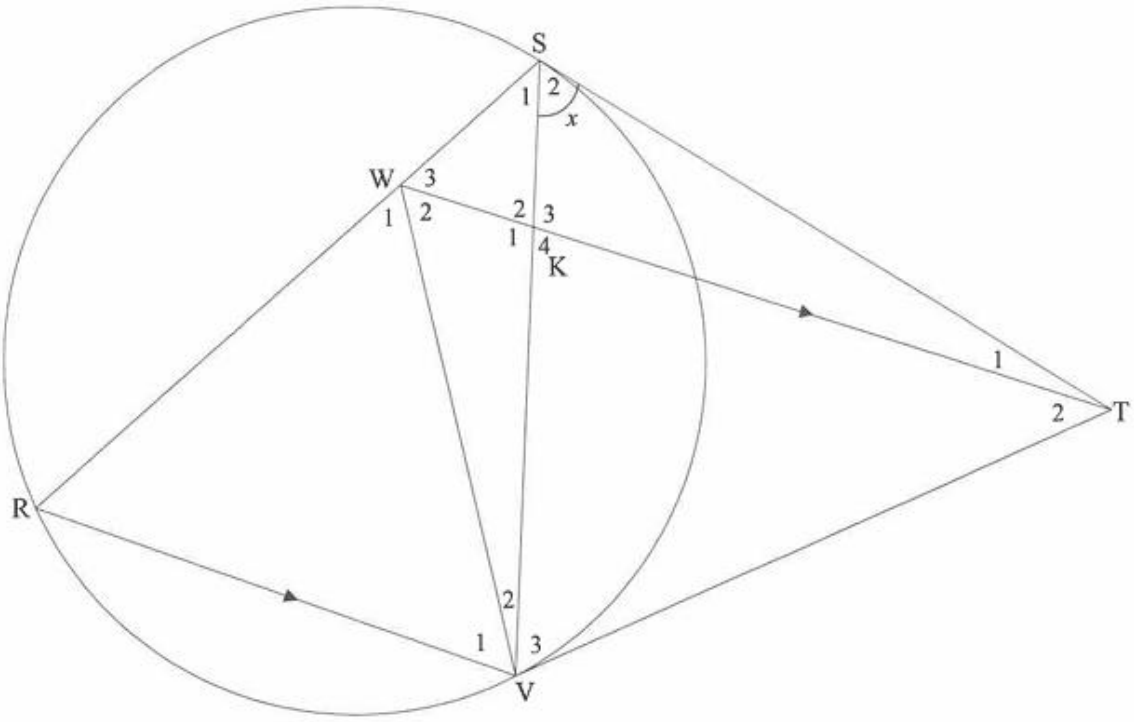
QUESTION 10

10.1 In the diagram, ΔABC is drawn. D is a point on AB and E is a point on AC such that DE || BC. BE and DC are drawn.



Use the diagram to prove the theorem which states that a line drawn parallel to one side of a triangle divides the other two sides proportionally, in other words prove that $\frac{AD}{DB} = \frac{AE}{EC}$ (6)

10.2 In the diagram, ST and VT are tangents to the circle at S and V respectively. R is a point on the circle and W is a point on chord RS such that WT is parallel to RV. SV and WV are drawn. WT intersects SV at K. Let $\hat{S}_2 = x$.



10.2.1 Write down, with reasons, THREE other angles EACH equal to x. (6)

10.2.2 Prove, with reasons, that:

(a) WSTV is a cyclic quadrilateral (2)

(b) ΔWRV is isosceles (4)

(c) $\Delta WRV \,|||\, \Delta TSV$ (3)

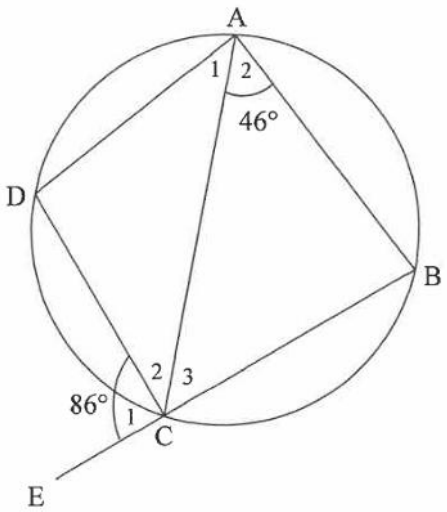
(d) $\frac{RV}{SR} = \frac{KV}{TS}$ (4)

[25]

Paper H

QUESTION 9

In the diagram, ABCD is a cyclic quadrilateral. BC is produced to E. AC is drawn.
$\hat{A}_1 = \frac{1}{2}\hat{B}$, $\hat{A}_2 = 46°$ and $\hat{C}_1 = 86°$.

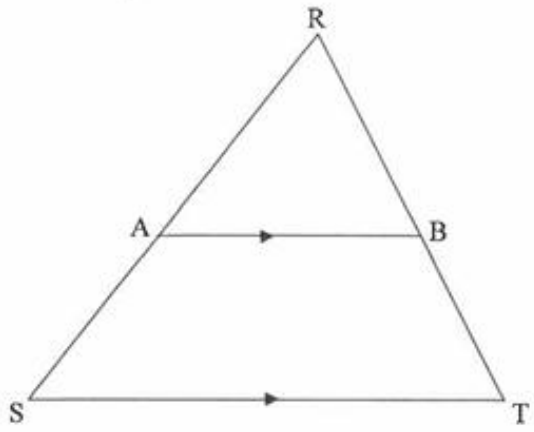


9.1 Calculate, with a reason, the value of $\hat{A}_1$. (2)

9.2 Hence, prove that AD = DC. (4)
[6]

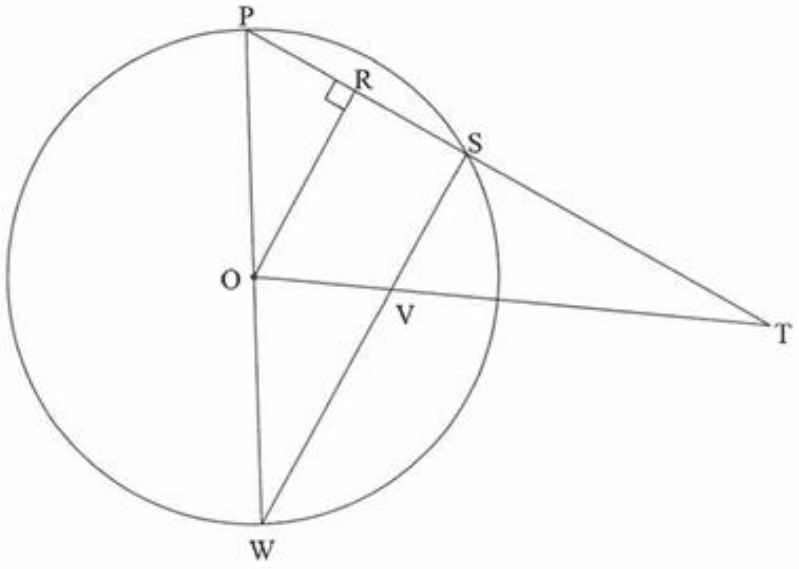
QUESTION 10

10.1 In the diagram, ΔRST is drawn. Line AB intersects RS and RT at A and B respectively such that AB $\parallel$ ST.



Prove the theorem which states that a line drawn parallel to one side of a triangle divides the other two sides proportionally, i.e. $\frac{RA}{AS} = \frac{RB}{BT}$ (6)

10.2 In the diagram, O is the centre of the circle. ΔPWS is drawn with P, W and S on the circle. OR $\perp$ PS. PRS is produced to T. SW and OT intersect at V. OV : OT = 1 : 4

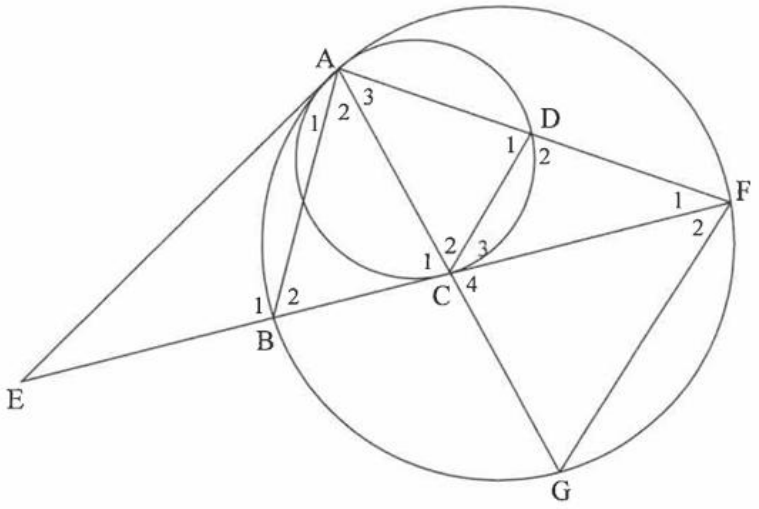


10.2.1 Prove, with reasons, that OR : WS = 1 : 2 (5)

10.2.2 Calculate the length of PT if ST = 15 units. (4)
[15]

QUESTION 11

In the diagram, A, B, G and F lie on the larger circle. A smaller circle is drawn to touch the larger circle internally at A. EA is a common tangent to both circles. EBCF is a tangent to the smaller circle at C. AC is produced to G. AF cuts the smaller circle at D. AB, CD and GF are drawn.



11.1 If $E\hat{A}G = x$, determine with reasons, FOUR other angles that are equal to x. (6)

11.2 Prove that AG.AD = AC.AF (4)

11.3 Prove that $\Delta AGF \,|||\, \Delta ABC$ (4)

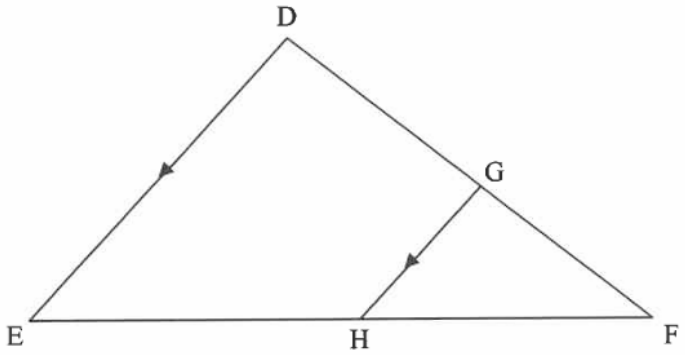
11.4 Prove that $GF^2 = \dfrac{BC.FC.AF}{AD}$ (6)

[20]

Paper I

QUESTION 9

9.1 In the diagram, ΔDEF is drawn. Line GH intersects DF and EF at G and H respectively such that $GH \parallel DE$ and $\frac{GF}{DG} = \frac{2}{5}$.



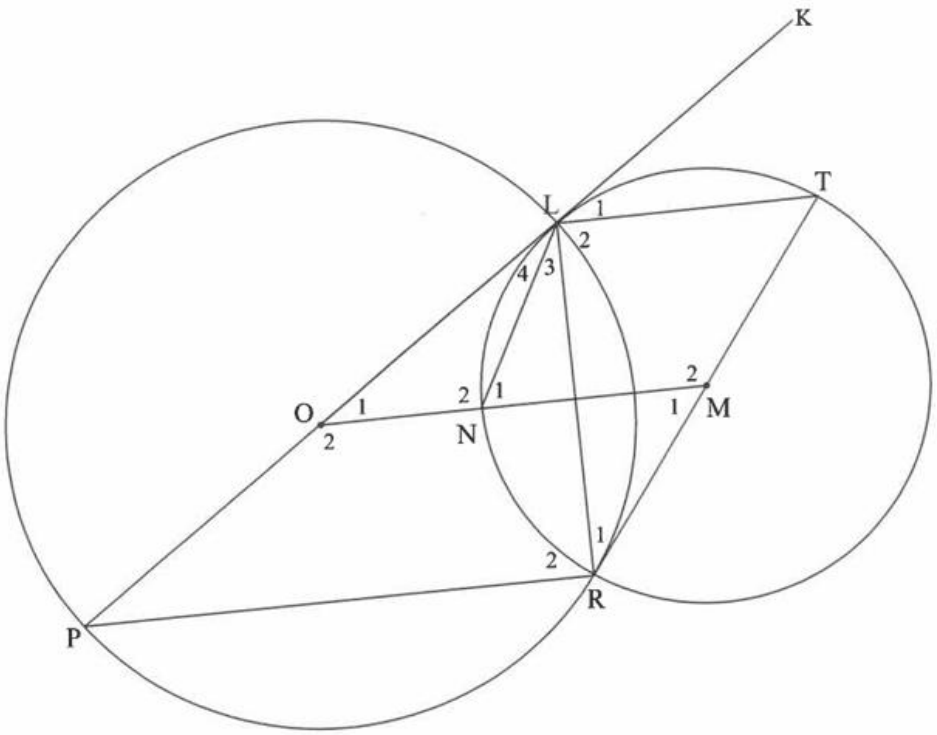
9.1.1 Write down, with a reason, the value of $\frac{HF}{EH}$. (2)

9.1.2 If EF = 21 cm, calculate the length of EH. (2)

9.1.3 Write down a triangle which is similar to ΔFGH. (1)

9.1.4 Hence, calculate the value of $\frac{GH}{DE}$. (2)

9.2 In the diagram, POL is a diameter of the larger circle with centre O. TMR is a diameter of the smaller circle with centre M. The two circles intersect at L and R. PLK is a tangent to the smaller circle at L and TR is a tangent to the larger circle at R. OM intersects the smaller circle at N. Straight lines LT, LR, LN and PR are drawn.



Prove, giving reasons, that:

9.2.1 LT || PR (4)

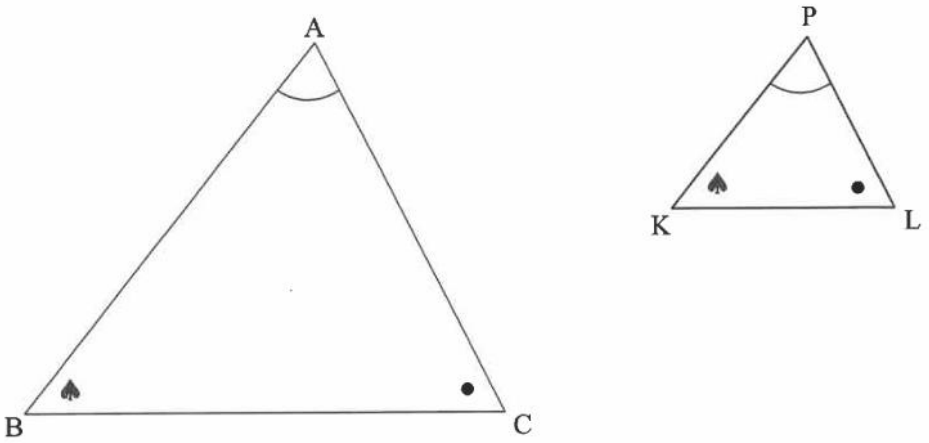
9.2.2 LORM is a cyclic quadrilateral, if it is also given that LT || OM (5)

9.2.3 LN bisects $O\hat{L}R$ (4)

[20]

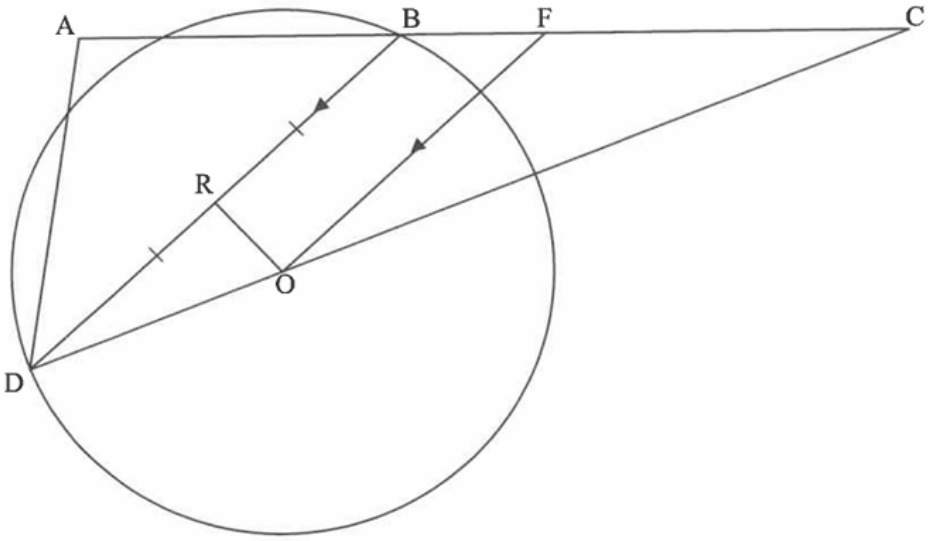
QUESTION 10

10.1 In the diagram, ΔABC and ΔPKL are drawn such that $\hat{A} = \hat{P}$, $\hat{B} = \hat{K}$ and $\hat{C} = \hat{L}$.



Use the diagram in the ANSWER BOOK to prove the theorem which states that if two triangles are equiangular, then the corresponding sides are in proportion, i.e. that $\frac{AB}{PK} = \frac{AC}{PL}$. (6)

10.2 In the diagram, O is the centre of the circle. Points D and B lie on the circle. Points A and C lie outside the circle such that side AC of ΔADC passes through B. F is a point on BC such that FO || BD. DR = RB and RO is drawn.



10.2.1 Prove, with reasons, that $\Delta CFO \;|||\; \Delta CBD$. (3)

10.2.2 If it is given that $R\hat{D}O = F\hat{C}O$, show, with reasons, that $OF.CD = CO.BC$ (2)

10.2.3 It is further given that DC = 19,2 units, BD = 12 units and $\frac{RO}{RD} = \frac{3}{4}$

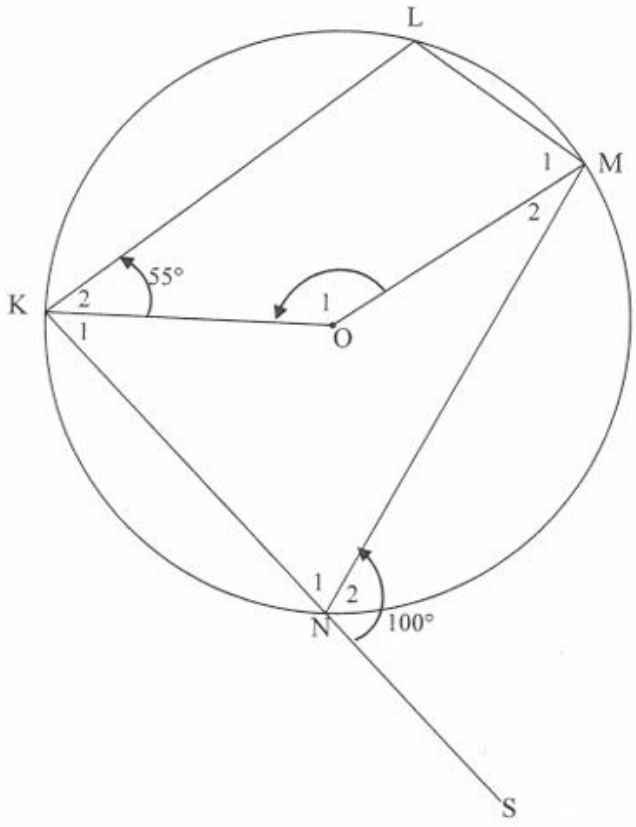
Prove, with reasons, that $BF = \frac{75}{16}$ (6)

10.2.4 Calculate the size of $A\hat{B}D$. (3)
[20]

Paper J

QUESTION 8

In the diagram, O is the centre of circle KLMN and KO and OM are joined. Chord KN is produced to S. $\hat{K}_2 = 55°$ and $\hat{N}_2 = 100°$.



Determine, with reasons, the size of the following:

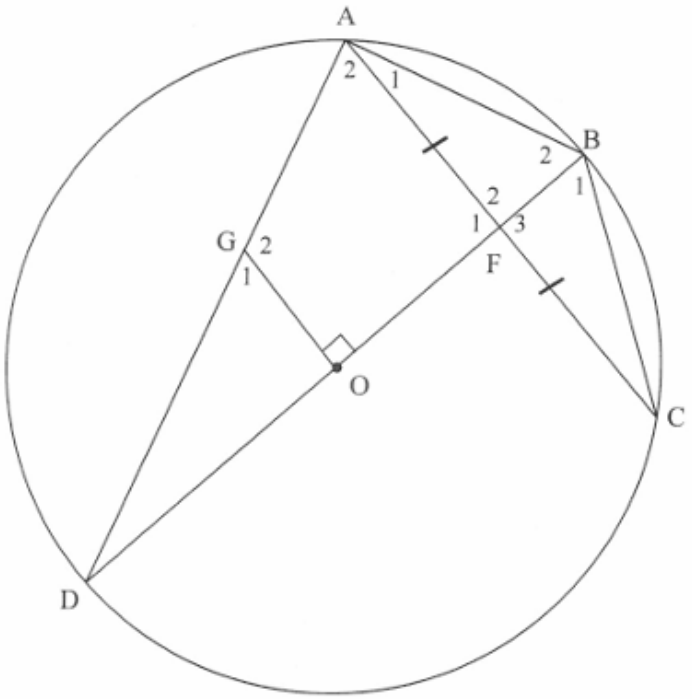
8.1 $\hat{L}$ (2)

8.2 $\hat{O}_1$ (3)

8.3 $\hat{M}_1$ (2)
[7]

QUESTION 9

In the diagram, O is the centre of circle ABCD and BOD is a diameter. F, the midpoint of chord AC, lies on BOD. G is a point on AD such that GO ⊥ DB.



9.1 Give a reason why:

9.1.1 $D\hat{A}B = 90°$ (1)

9.1.2 AGOB is a cyclic quadrilateral (1)

9.2 Prove that:

9.2.1 AC ∥ GO (3)

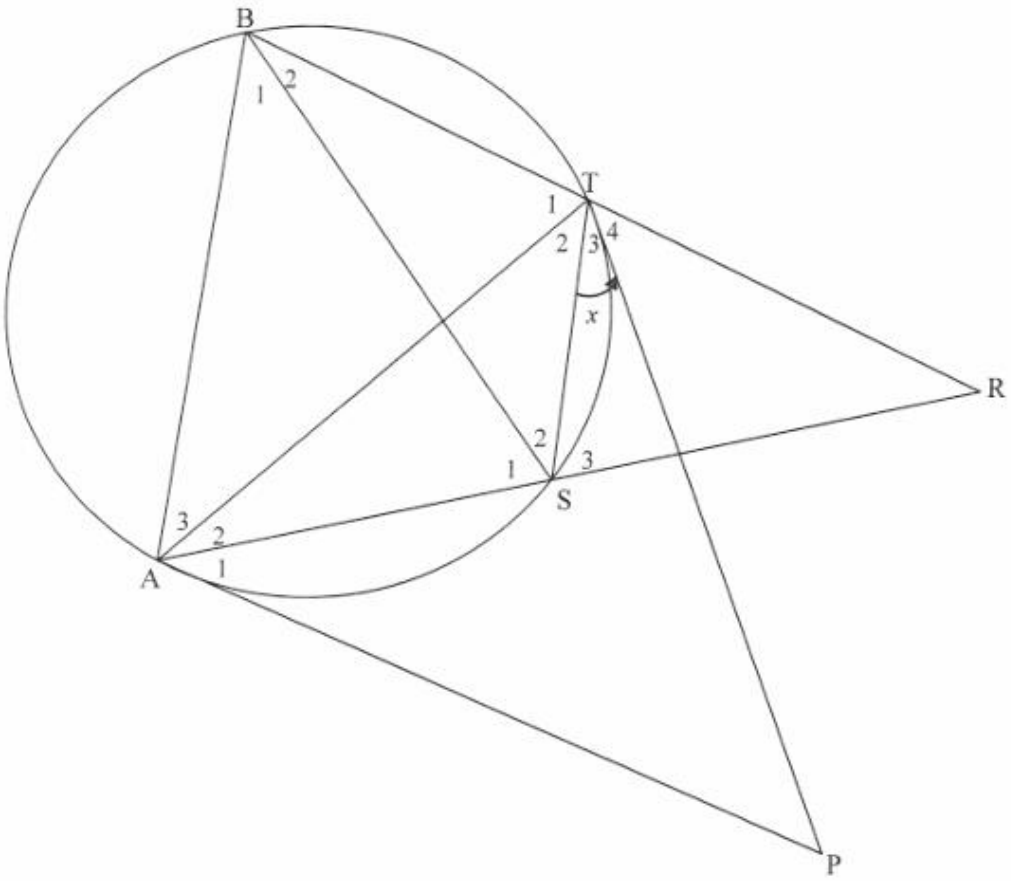
9.2.2 $\hat{G}_1 = \hat{B}_1$ (4)

9.3 If it is given that $FB = \frac{2}{5}r$, where r is the radius of the circle, determine, with reasons, the ratio of $\frac{DG}{DA}$. (3)

[12]

QUESTION 10

In the diagram, PA and PT are tangents to a circle at A and T respectively. B and S are points on the circle such that BT produced and AS produced meet at R and BR = AR. BS, AT and TS are drawn. $\hat{T}_3 = x$.



10.1 Give a reason why $\hat{T}_3 = \hat{A}_2 = x$. (1)

10.2 Prove that:

10.2.1 AB ∥ ST (5)

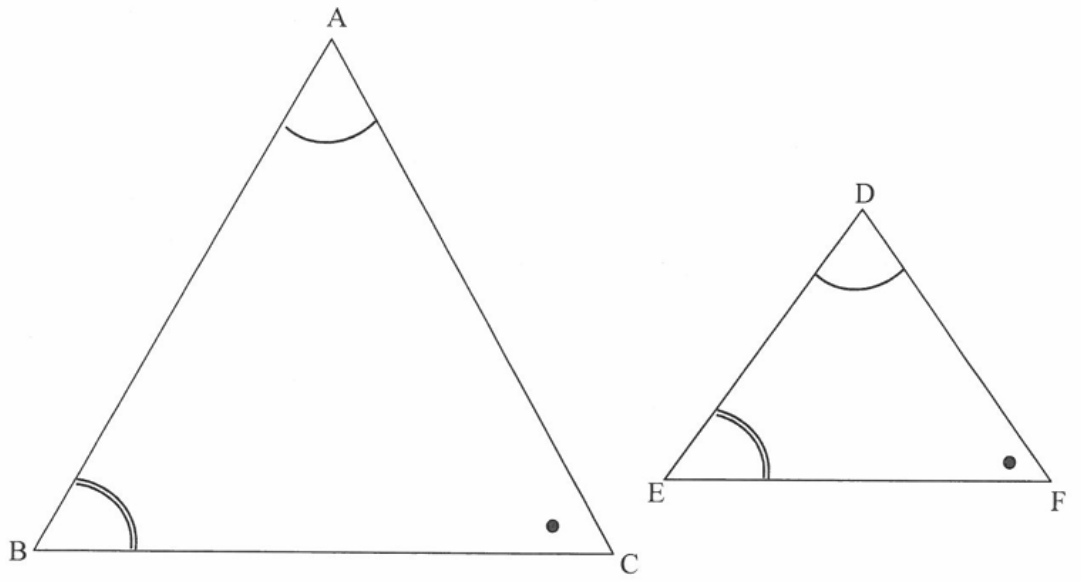
10.2.2 $\hat{T}_4 = \hat{A}_1$ (5)

10.2.3 RTAP is a cyclic quadrilateral (2)

[13]

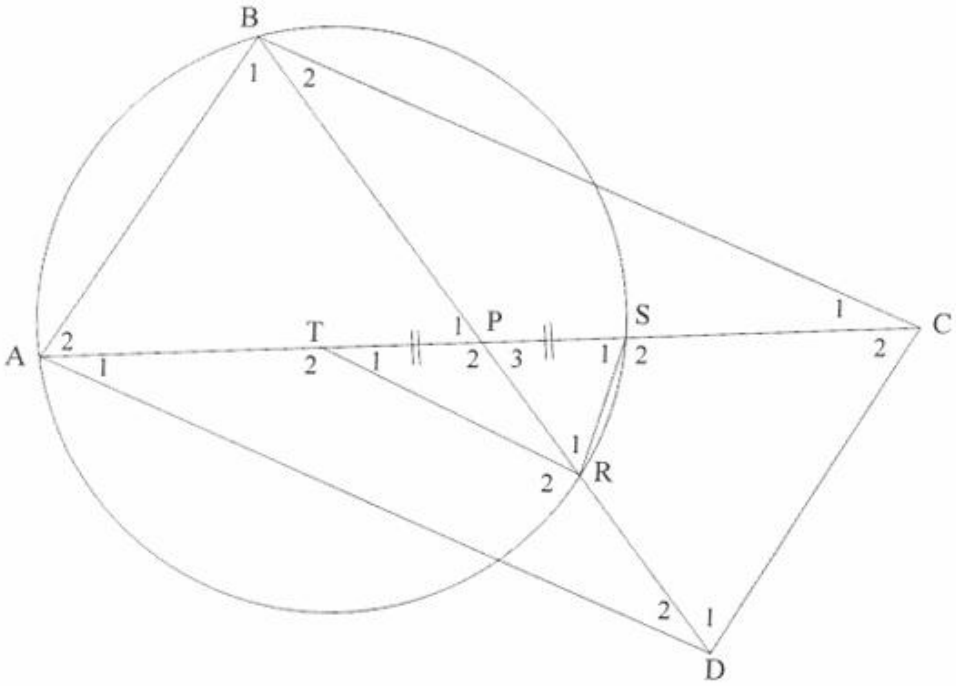
QUESTION 11

11.1 In the diagram, ΔABC and ΔDEF are drawn with $\hat{A} = \hat{D}$, $\hat{B} = \hat{E}$ and $\hat{C} = \hat{F}$.



Prove the theorem which states that if two triangles, ΔABC and ΔDEF, are equiangular, then $\frac{DE}{AB} = \frac{DF}{AC}$. (6)

11.2 In the diagram, ABCD is a parallelogram with A and B on the circle. The diagonals BD and AC intersect in P. PC and PD intersect the circle at S and R respectively. T is a point on AP such that TP = PS. TR is drawn.



11.2.1 Prove that:

(a) AT = SC (2)

(b) ΔPSR ||| ΔPBA (5)

11.2.2 If it is further given that $\frac{PR}{PA} = \frac{TR}{AD}$, prove that:

(a) ΔRPT ||| ΔAPD (3)

(b) ATRD is a cyclic quadrilateral (2)

[18]

SECTION 3: ANSWER BOOK SECTIONS

STATISTICS

Paper B

2.1

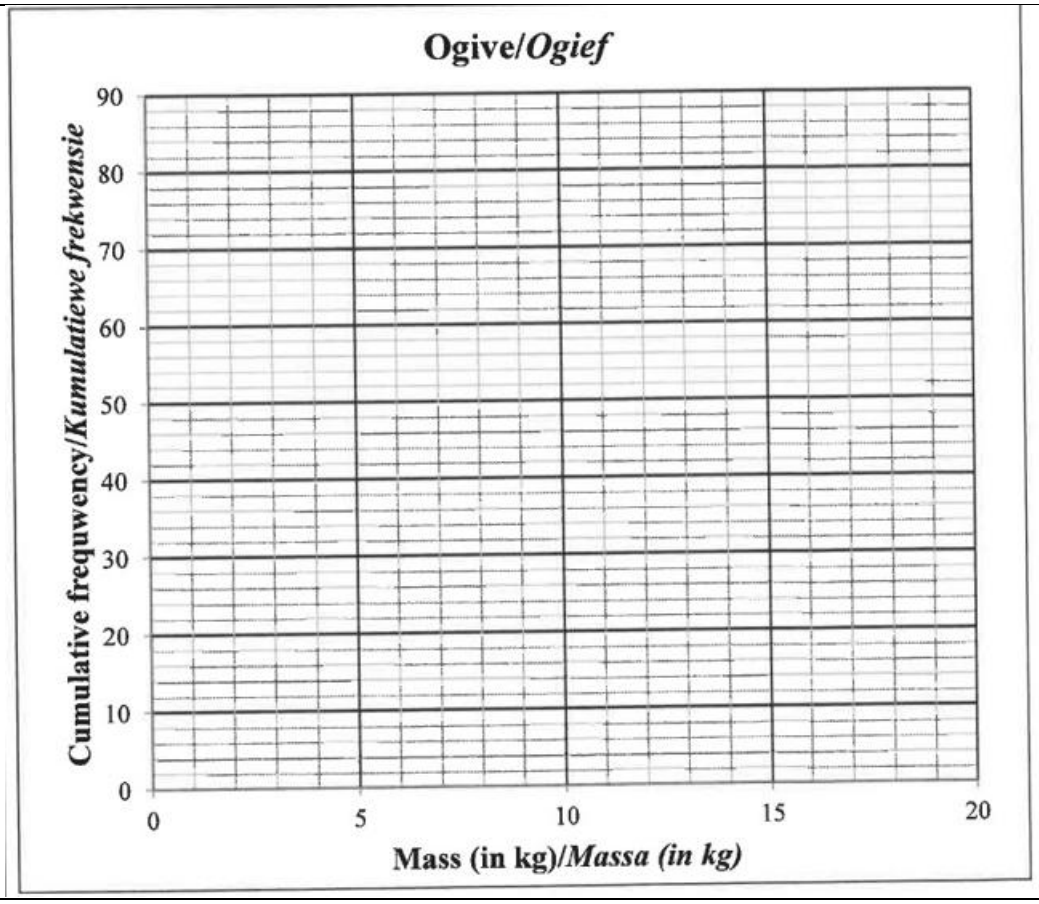
Number of glasses of water drank per day/ *Aantal glase water per dag gedrink*	Number of staff members/ *Getal personeellede*	Cumulative frequency/ *Kumulatiewe frekwensie*
$0 \leq x < 2$	5	
$2 \leq x < 4$	15	
$4 \leq x < 6$	13	
$6 \leq x < 8$	5	
$8 \leq x < 10$	2	

Paper C

1.2

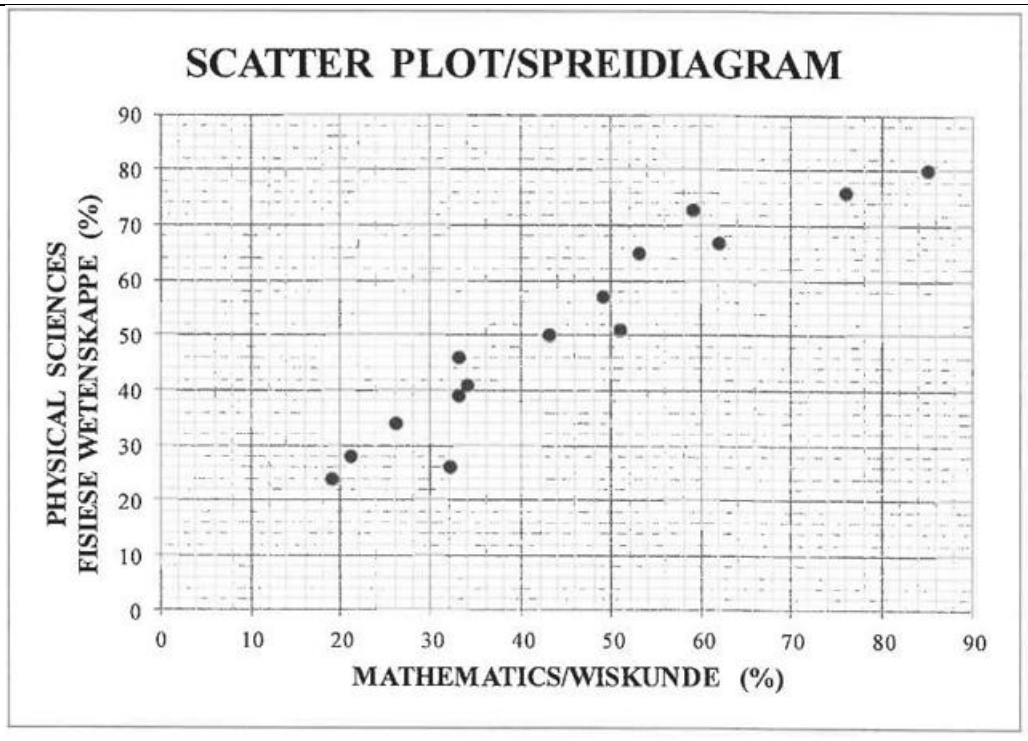
MASS (in kg)/ *MASSA (in kg)*	FREQUENCY/ *FREKWENSIE*	CUMULATIVE FREQUENCY/ *KUMULATIEWE FREKWENSIE*
$5 < m \leq 7$	6	
$7 < m \leq 9$	18	
$9 < m \leq 11$	21	
$11 < m \leq 13$	19	
$13 < m \leq 15$	11	
$15 < m \leq 17$	4	
$17 < m \leq 19$	1	

1.2


Ogive/*Ogief*
Cumulative frequency/*Kumulatiewe frekwensie*
0
10
20
30
40
50
60
70
80
90
0
5
10
15
20
Mass (in kg)/*Massa (in kg)*

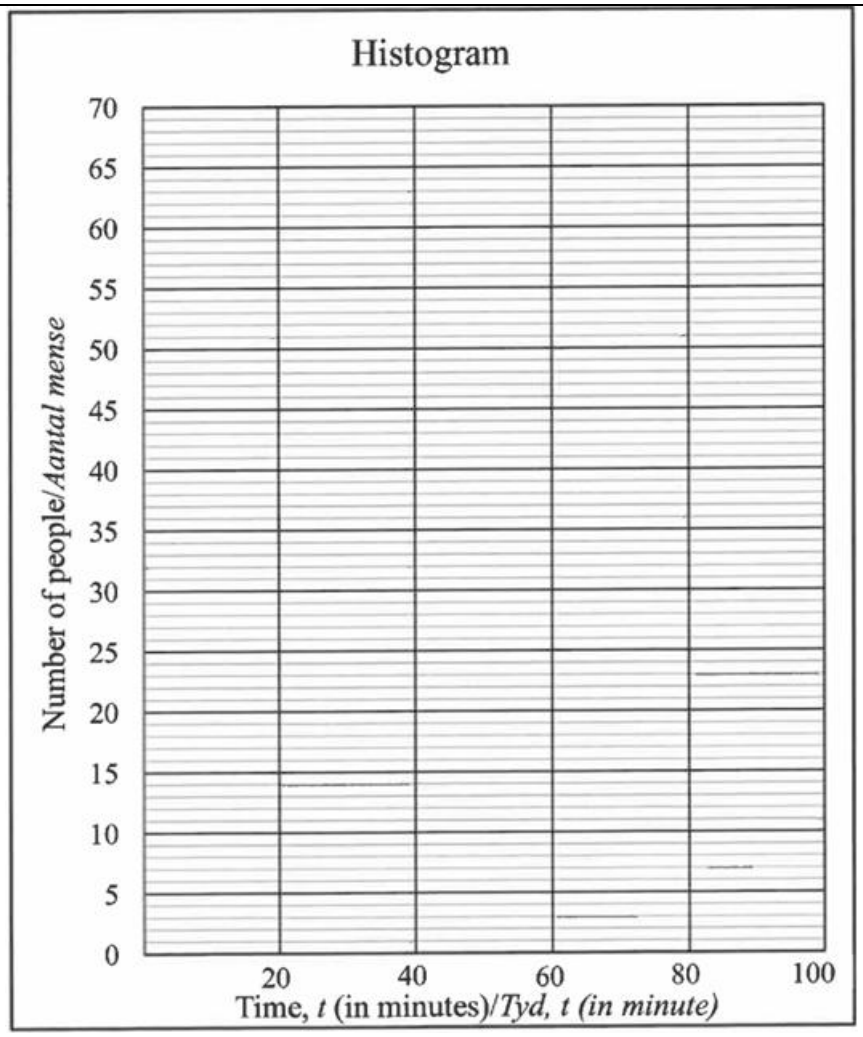
Paper D

1.2


SCATTER PLOT/SPREIDIAGRAM
PHYSICAL SCIENCES
FISIESE WETENSKAPPE (%)
0
10
20
30
40
50
60
70
80
90
0
10
20
30
40
50
60
70
80
90
MATHEMATICS/WISKUNDE (%)

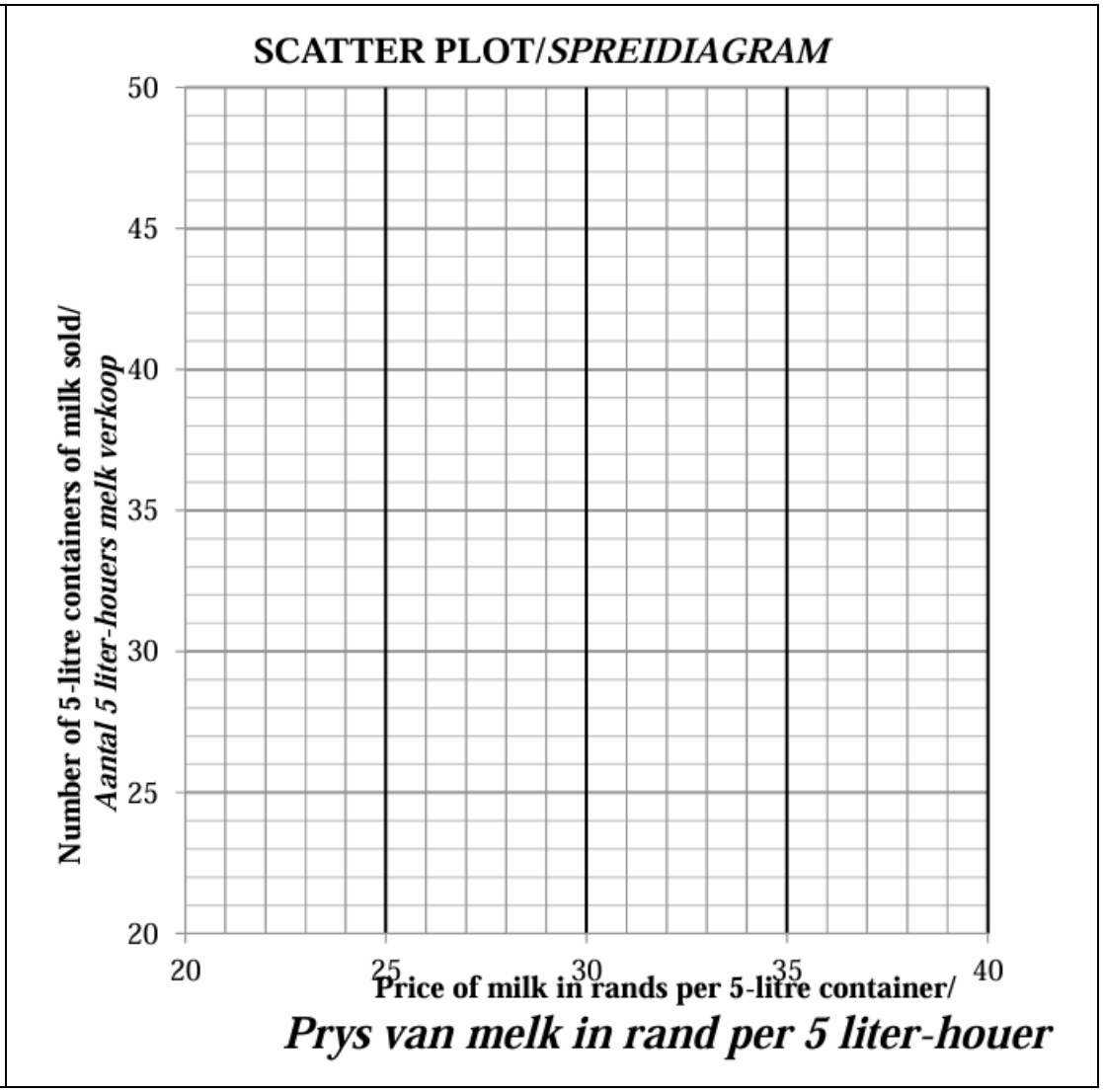
Paper E

2.1.3


Histogram
70
65
60
55
50
45
40
35
30
25
20
15
10
5
0
Number of people/Aantal mense
20
40
60
80
100
Time, t (in minutes)/Tyd, t (in minute)

Paper G

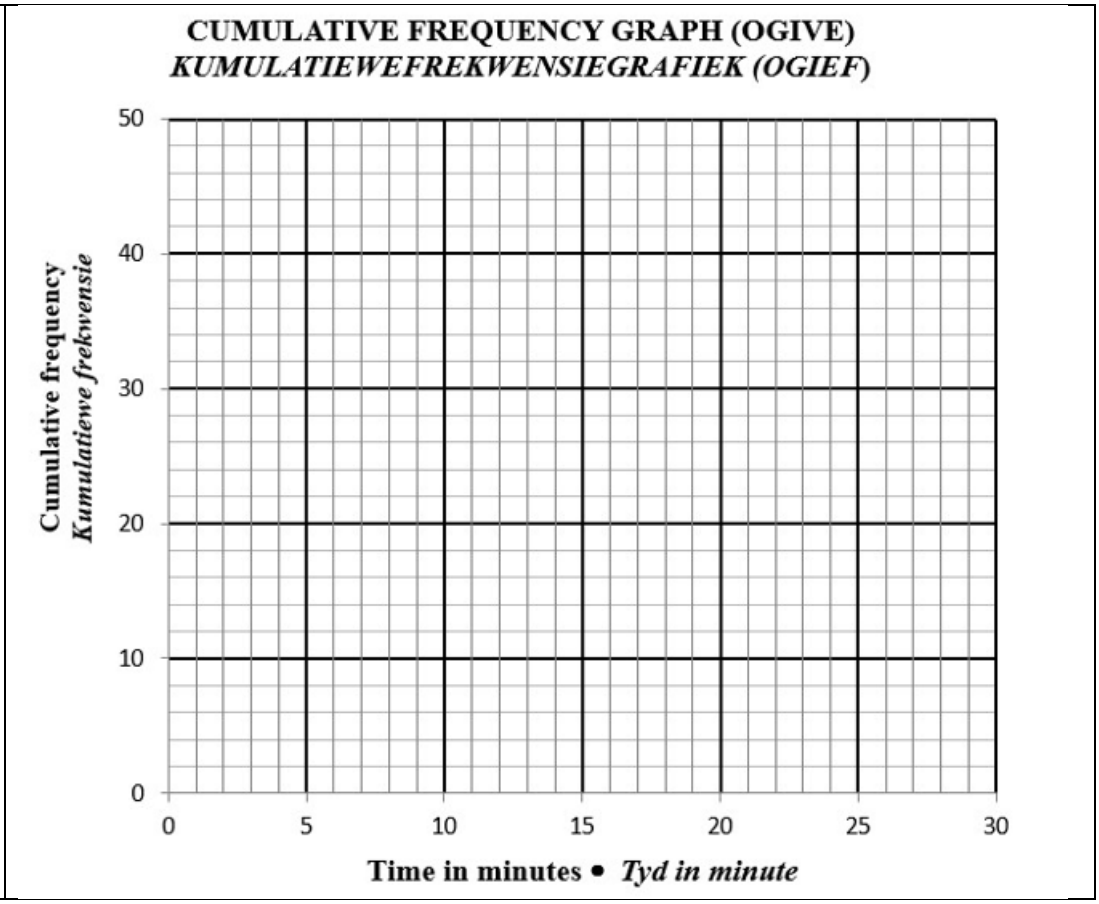
2.1



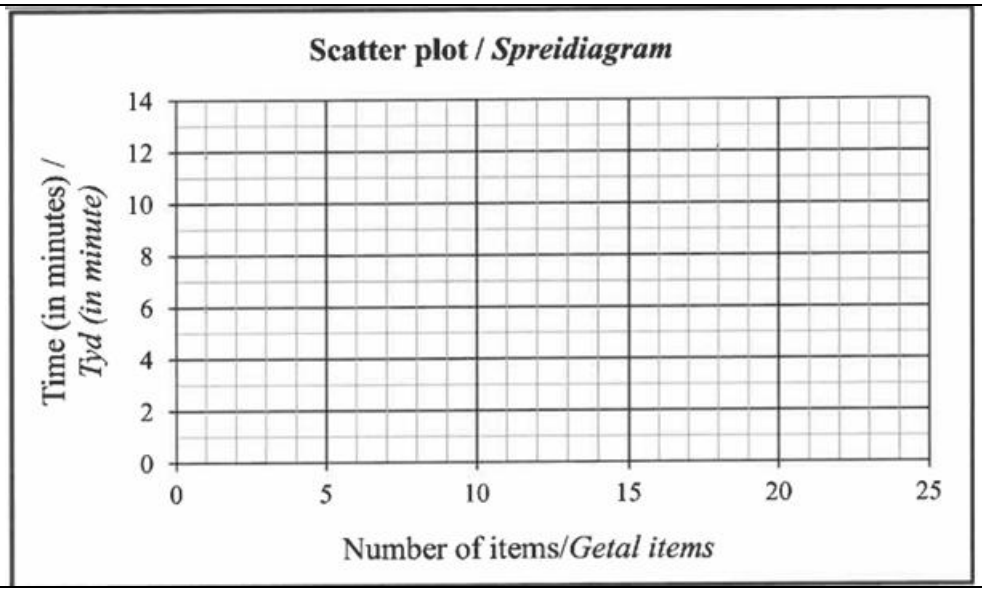
Paper H

1.3

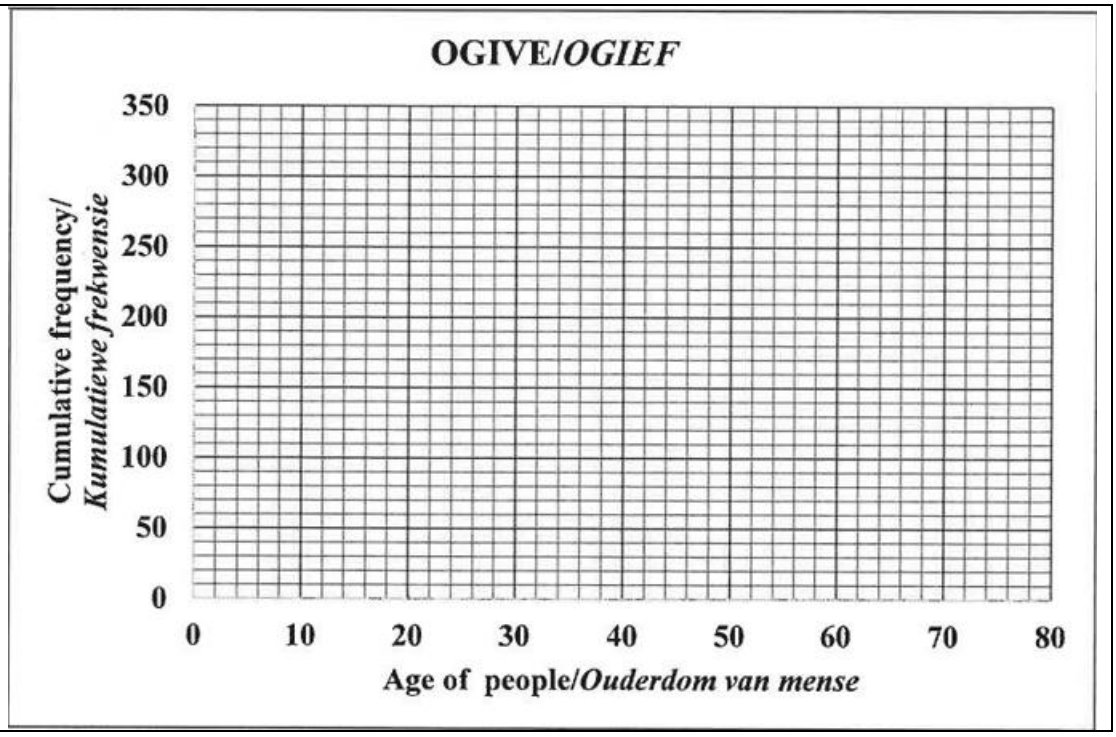
Time in minutes (t) *Tyd in minute (t)*	Number of children *Getal kinders*	Cumulative frequency *Kumulatiewe frekwensie*
$2 < t \leq 6$	2	
$6 < t \leq 10$	10	
$10 < t \leq 14$	9	
$14 < t \leq 18$	7	
$18 < t \leq 22$	8	
$22 < t \leq 26$	7	
$26 < t \leq 30$	2	

1.4	CUMULATIVE FREQUENCY GRAPH (OGIVE) KUMULATIEWEFREKWENSIEGRAFIEK (OGIEF) Cumulative frequency Kumulatiewe frekwensie 50 40 30 20 10 0 0 5 10 15 20 25 30 Time in minutes • Tyd in minute

Paper I

2.1	Scatter plot / Spreidiagram Time (in minutes) / Tyd (in minute) 14 12 10 8 6 4 2 0 0 5 10 15 20 25 Number of items/Getal items

Paper J

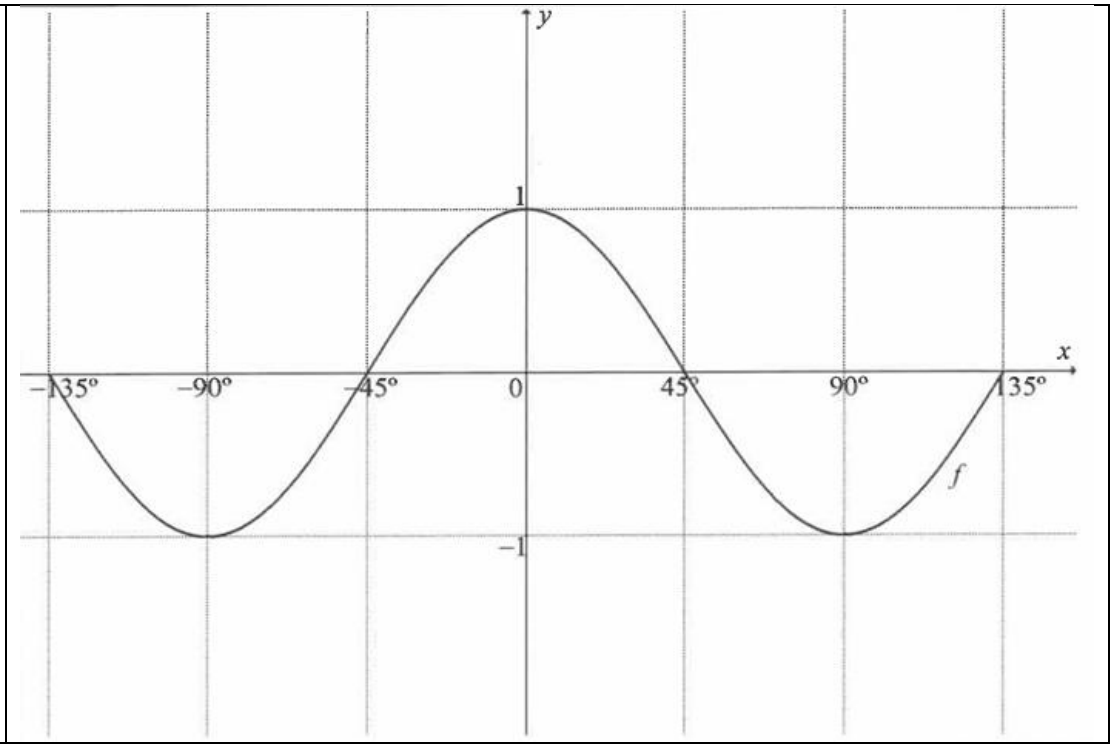
2.3	

OGIVE/*OGIEF*
Cumulative frequency/
Kumulatiewe frekwensie
350
300
250
200
150
100
50
0
0
10
20
30
40
50
60
70
80
Age of people/*Ouderdom van mense*

TRIGONOMETRY

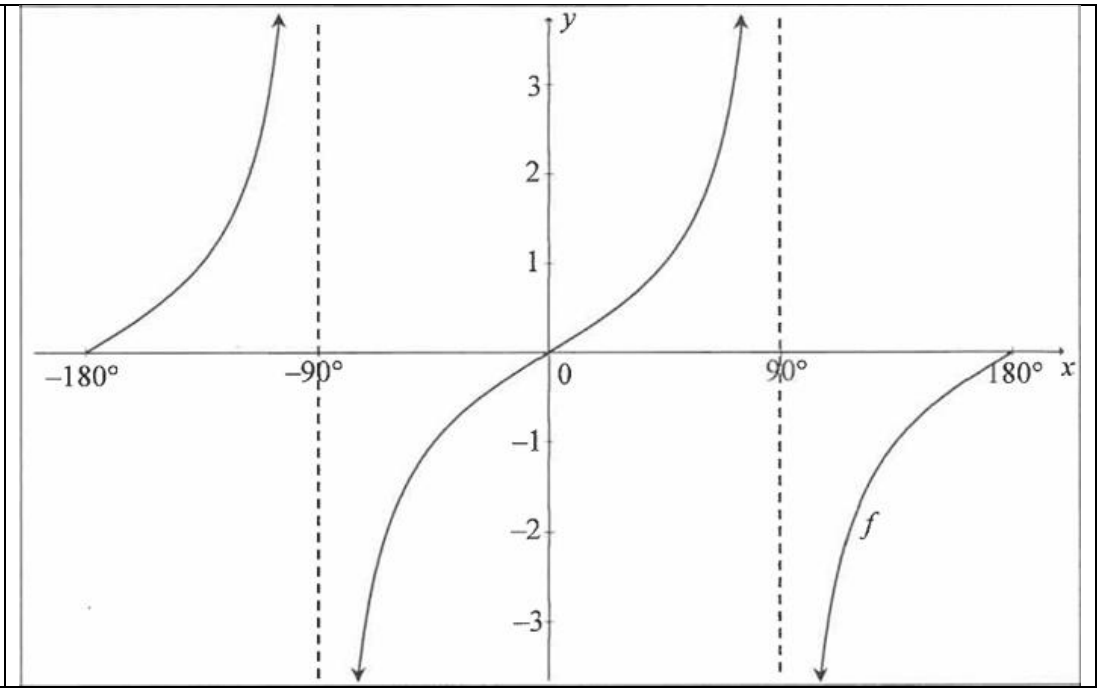
Paper D

7.2

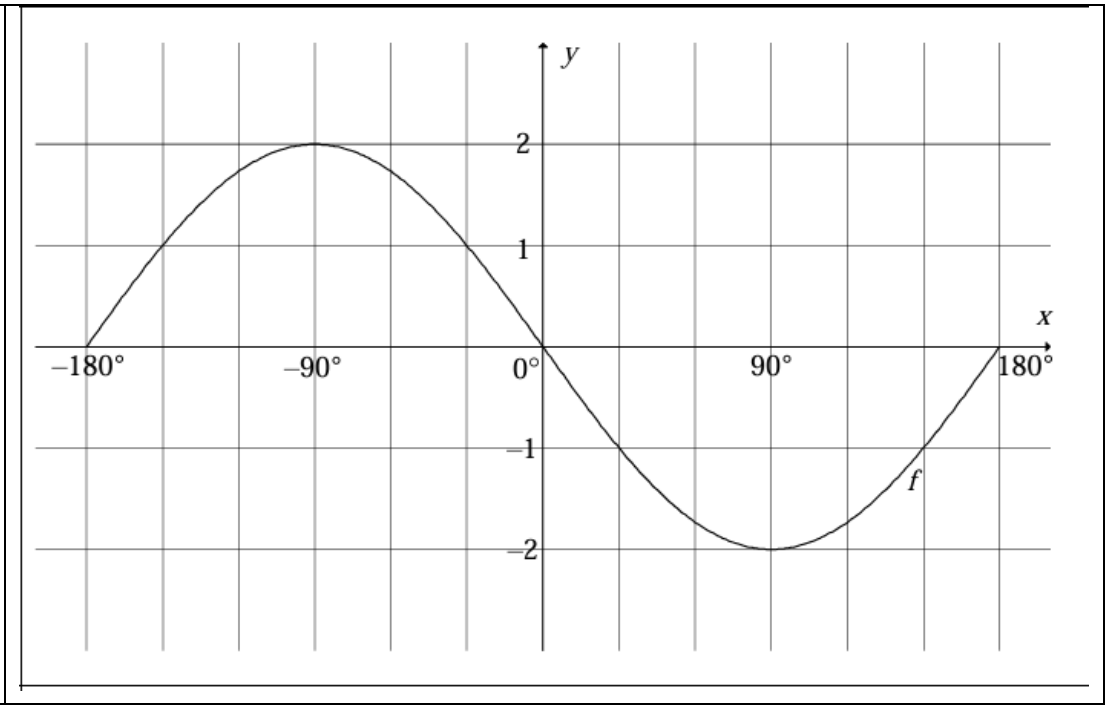


Paper F

7.3.2

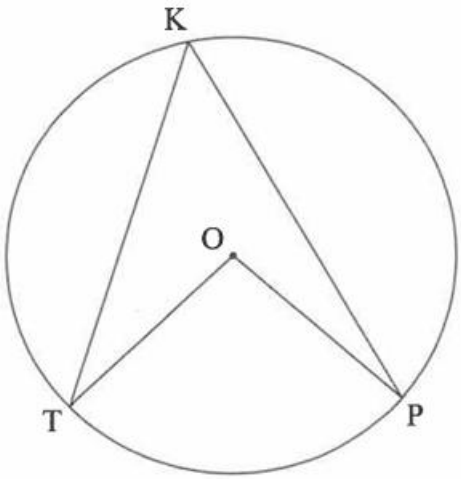


Paper H

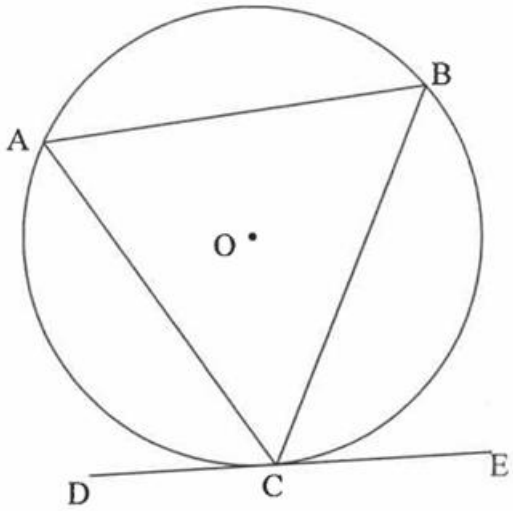
7.1	

EUCLIDEAN GEOMETRY

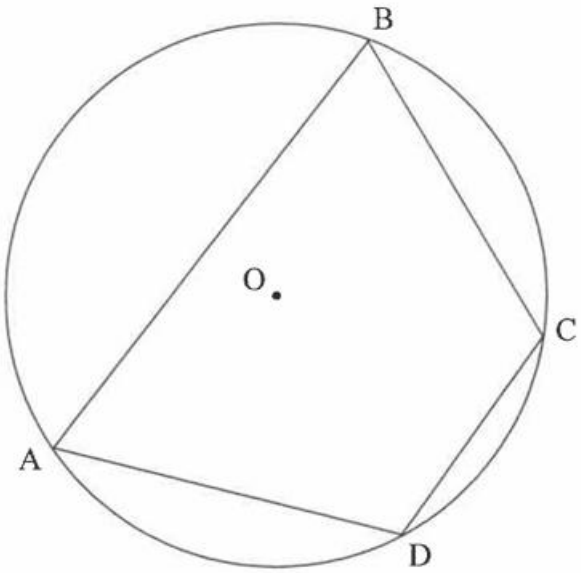
Paper A

9.1	

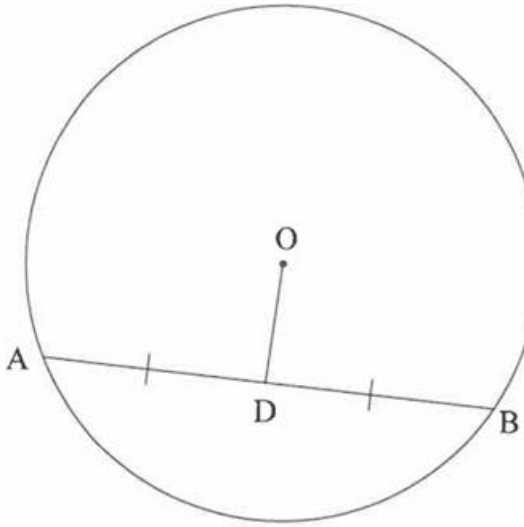
Paper B

8.1	

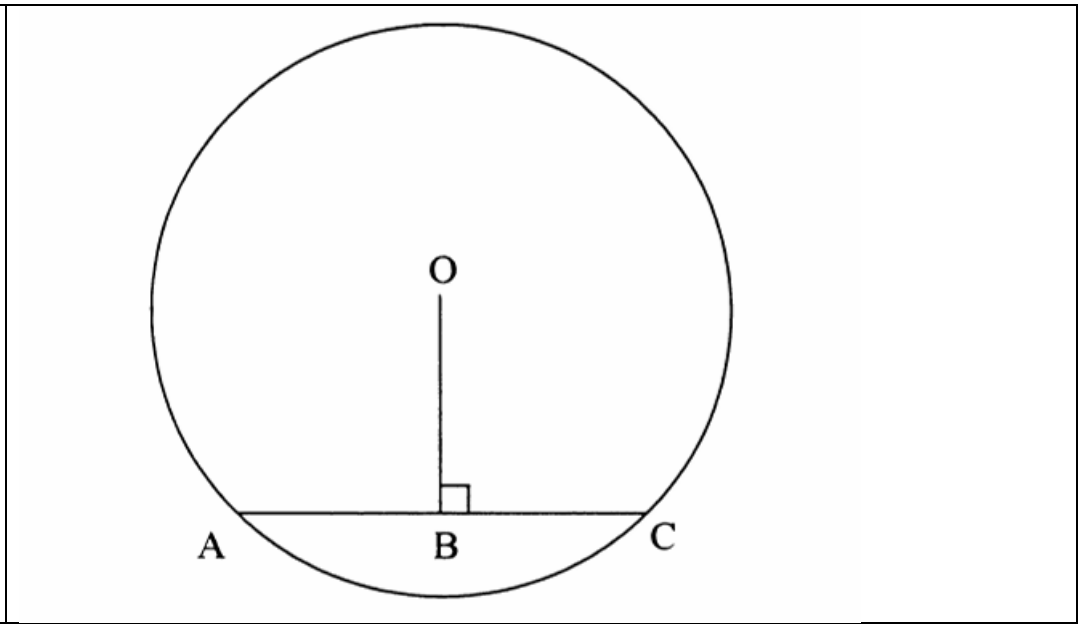
Paper C

9.1	

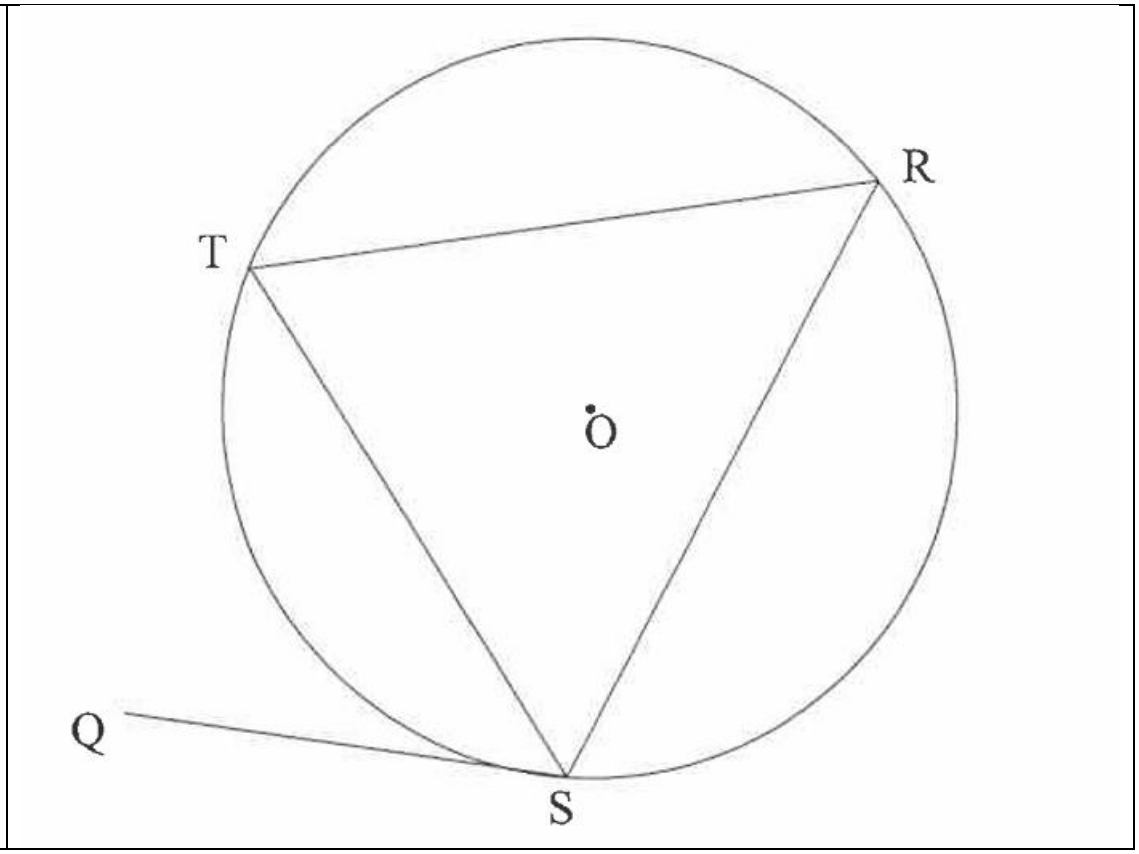
Paper D

9.1	

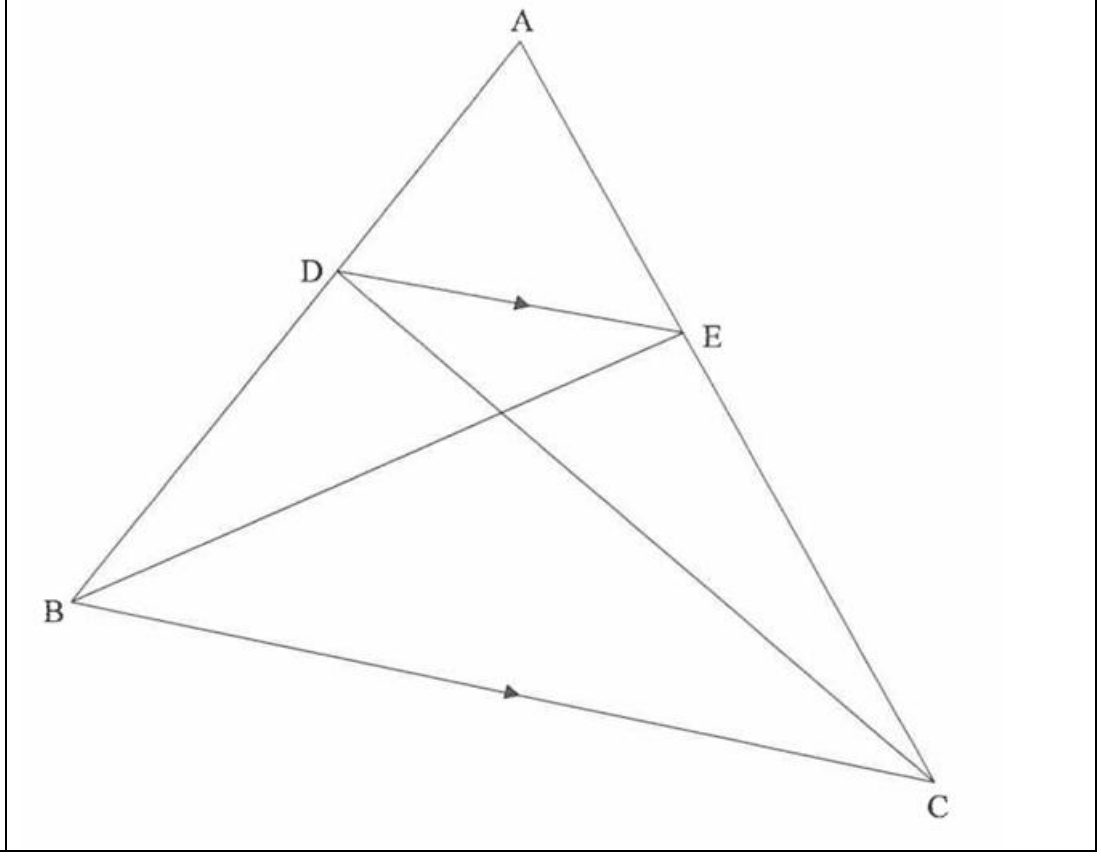
Paper E

11	

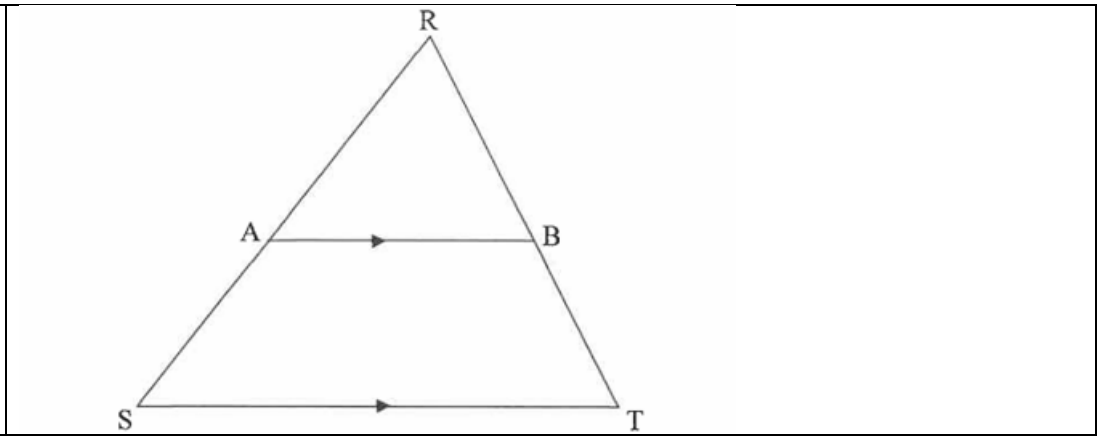
Paper F

9.1	

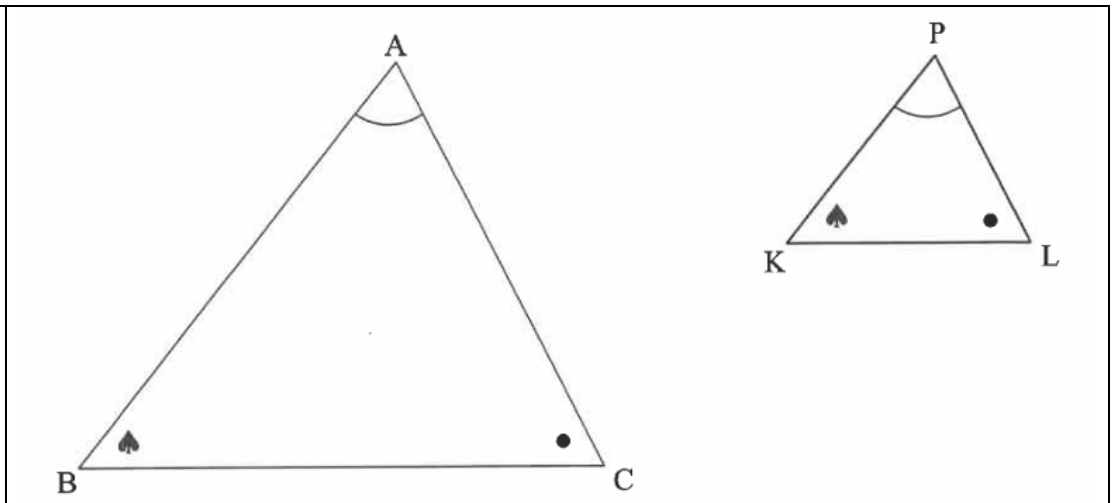
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10.1	

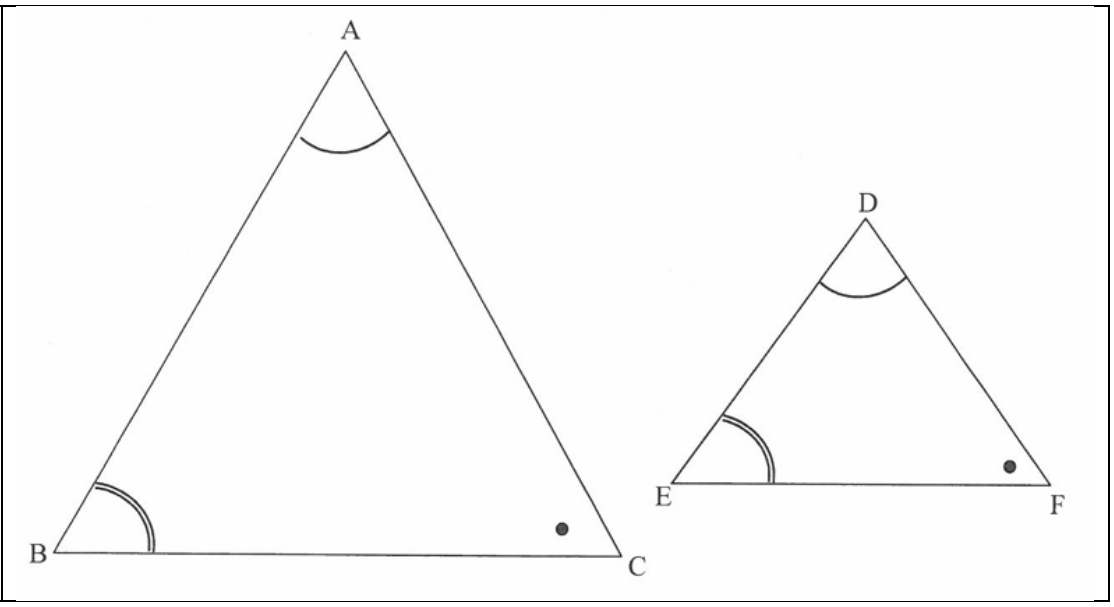
Paper H

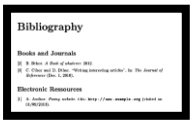
10.1	

Paper I

10.1	

Paper J

11.1	



BIBLIOGRAPHY

1	Past Exam Papers (2013 – 2026)